Expressions, Equations, and Inequalities

MODULE 6

Expressions and Equations

FL 7.EE.1.1, 7.EE.1.2, 7.EE.2.4, 7.EE.2.4a

MODULE 7

Inequalities

FL 7.EE.2.4, 7.EE.2.4b

CAREERS IN MATH

Mechanical Engineer A mechanical engineer designs, develops, and manufactures mechanical devices and technological systems. Mechanical engineers use math to solve diverse problems, from calculating the strength of materials to determining energy consumption of a device.

If you are interested in a career in mechanical engineering, you should study these mathematical subjects:
- Algebra
- Geometry
- Trigonometry
- Statistics
- Calculus

Research other careers that require the daily use of mathematics to solve problems.

Unit 3 Performance Task

At the end of the unit, check out how **mechanical engineers** use math.

Vocabulary Preview

Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters to answer the riddle at the bottom of the page.

An expression that contains at least one variable. **(Lesson 6.1)**

___ ___ ___ ___ ___ ___ ___

___ ___ ___ ___ ___ ___ ___

An equation with more than one operation. **(Lesson 6.3)**

___ ___ ___   ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

A variable whose value is less than zero. **(Lesson 6.3)**

___ ___ ___ ___ ___ ___ ___   ___ ___ ___ ___ ___ ___ ___

A variable whose value is greater than zero. **(Lesson 6.3)**

___ ___ ___ ___ ___ ___   ___ ___ ___ ___ ___ ___ ___

A mathematical sentence that shows the relationship between quantities that are not equivalent. **(Lesson 7.1)**

___ ___ ___ ___ ___ ___ ___ ___

Q: Why does the sum of −4 and 3 complain more than the sum of −3 and 5?

A: It's the ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___!

Expressions and Equations

? ESSENTIAL QUESTION

How can you use algebraic expressions and equations to solve real-world problems?

Real-World Video

When you take a taxi, you will be charged an initial fee plus a charge per mile. To describe situations like this, you can write a two-step equation.

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Are YOU Ready?

Complete these exercises to review skills you will need for this chapter.

Words for Operations

EXAMPLE   the difference of 2 and b *Difference means subtraction.*

$2 - b$

the product of -8 and a number   *Product means multiplication.*

$(-8)x$ or $-8x$ *Let x represent the unknown number.*

Write an algebraic expression for each word expression.

1. the sum of 5 and a number x _____

2. 11 decreased by n _____

3. the quotient of -9 and y _____

4. twice a number, minus 13 _____

Evaluate Expressions

EXAMPLE   Evaluate $3x - 5$ for $x = -2$.

$3x - 5 = 3(-2) - 5$ *Substitute the given value of x for x.*

$= -6 - 5$ *Multiply.*

$= -11$ *Subtract.*

Evaluate each expression for the given value of x.

5. $2x + 3$ for $x = 3$ _____

6. $-4x + 7$ for $x = -1$ ___

7. $1.5x - 2.5$ for $x = 3$ _____

8. $0.4x + 6.1$ for $x = -5$ ___

9. $\frac{2}{3}x - 12$ for $x = 18$ ____

10. $-\frac{5}{8}x + 10$ for $x = -8$ ____

Operations with Fractions

EXAMPLE $\frac{2}{5} \div \frac{7}{10}$ $\frac{2}{5} \div \frac{7}{10} = \frac{2}{5} \times \frac{10}{7}$ *Multiply by the reciprocal of the divisor.*

$= \frac{2}{\underset{1}{5}} \times \frac{\overset{2}{10}}{7}$ *Divide by the common factors.*

$= \frac{4}{7}$ *Simplify.*
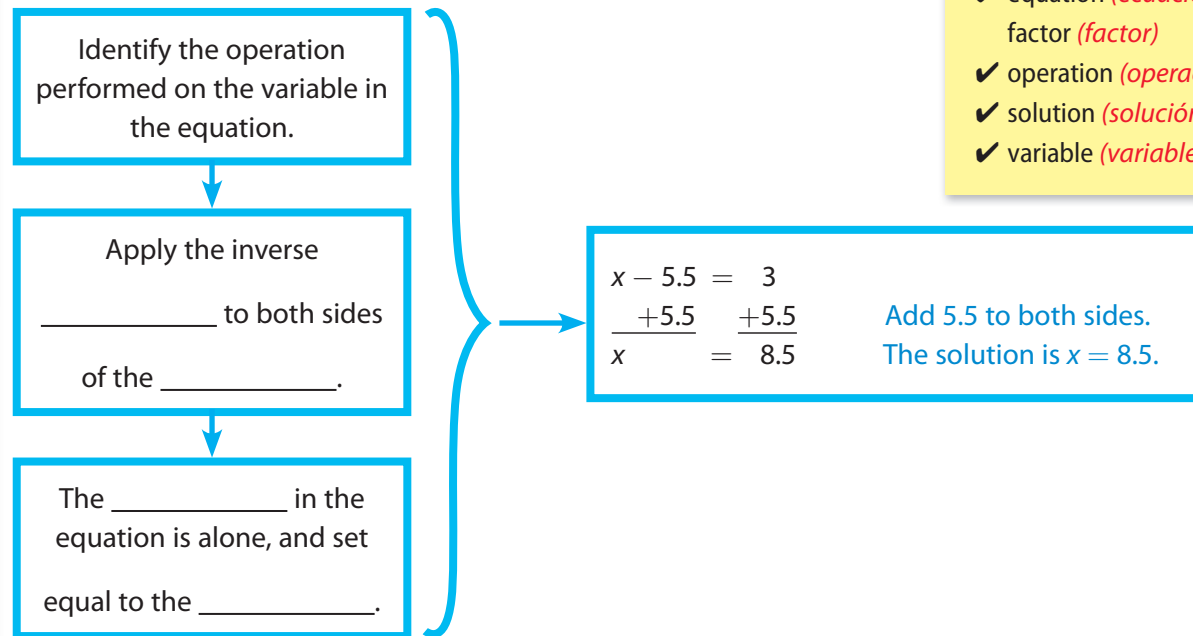
Divide.

11. $\frac{1}{2} \div \frac{1}{4}$ _____

12. $\frac{3}{8} \div \frac{13}{16}$ _____

13. $\frac{2}{5} \div \frac{14}{15}$ _____

14. $\frac{4}{9} \div \frac{16}{27}$ _____

Reading **Start-Up**

Visualize Vocabulary

Use the ✔ words to complete the graphic. You may put more than one word in each box.

Identify the operation performed on the variable in the equation.

↓

Apply the inverse _____ to both sides of the _____.

↓

The _____ in the equation is alone, and set equal to the _____.

$$x - 5.5 = 3$$
$$\underline{+5.5 \quad +5.5}$$
$$x \quad = \quad 8.5$$

Add 5.5 to both sides.
The solution is $x = 8.5$.

Vocabulary

Review Words

   algebraic expression *(expresión algebraica)*
   Distributive Property *(Propiedad distributiva)*
✔ equation *(ecuación)*
   factor *(factor)*
✔ operation *(operación)*
✔ solution *(solución)*
✔ variable *(variable)*

Understand Vocabulary

Complete the sentences using the review words.

1. A(n) _____ contains at least one variable.

2. A mathematical sentence that shows that two expressions are equivalent

   is called a(n) _____.

Active Reading

Tri-Fold Before beginning the module, create a tri-fold to help you learn the concepts and vocabulary in this module. Fold the paper into three sections. Label the columns "What I Know," "What I Need to Know," and "What I Learned." Complete the first two columns before you read. After studying the module, complete the third column.

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

FL 7.EE.1.1

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Key Vocabulary

coefficient *(coeficiente)*
The number that is multiplied by the variable in an algebraic expression.

rational number *(número racional)* Any number that can be expressed as a ratio of two integers.

What It Means to You

You will use your knowledge of properties of operations to write equivalent expressions.

UNPACKING EXAMPLE 7.EE.1.1

Expand the expression $2(a + 7)$ using the distributive property.

$$2(a + 7) = 2 \cdot a + 2 \cdot 7 \quad \text{Multiply each term in parentheses by 2.}$$
$$= 2a + 14$$

FL 7.EE.2.4a

Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently.

Key Vocabulary

equation *(ecuación)*
A mathematical sentence that shows that two expressions are equivalent.

solution *(solución)*
The value for the variable that makes the equation true.

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What It Means to You

You will write and solve real-world equations that require two steps.

UNPACKING EXAMPLE 7.EE.2.4a

Jai and Lúpe plan to rent a kayak. The rental is $12 for the first hour and $9 for each hour after that. If they have $50, for how long can they rent the kayak?

Rental Charge $= 12 + 9x$, where x is the number of hours after the first hour.

$$50 = 12 + 9x$$
$$50 - 12 = 12 - 12 + 9x \quad \text{Subtract 12 from both sides.}$$
$$38 = 9x$$
$$\frac{38}{9} = x, \text{ or } x \approx 4.2 \quad \text{Divide both sides by 9.}$$

They can rent the kayak for 4 hours.

LESSON
6.1 Algebraic Expressions

? ESSENTIAL QUESTION

How do you add, subtract, factor, and multiply algebraic expressions?

Adding and Subtracting Expressions

You can use the properties of addition along with the Distributive Property to add and subtract algebraic expressions.

EXAMPLE 1 Real World   FL 7.EE.1.1, 7.EE.1.2

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Jill and Kyle get paid per project. Jill is paid a project fee of $25 plus $10 per hour. Kyle is paid a project fee of $18 plus $14 per hour. Write an expression to represent how much a company will pay to hire both to work the same number of hours on a project.

STEP 1 Write expressions for how much the company will pay each person. Let h represent the number of hours they will work on the project.

Jill: $\$25 + \$10h$ Kyle: $\$18 + \$14h$

Fee + Hourly rate × Hours   Fee + Hourly rate × Hours

STEP 2 Add the expressions to represent the amount the company will pay to hire both.

$25 + 10h + 18 + 14h$ *Combine their pay.*

$= 25 + 18 + 10h + 14h$ *Use the Commutative Property.*

$= 43 + 24h$ *Combine like terms.*

The company will pay $43 + 24h$ dollars to hire both Jill and Kyle.

Reflect

1. **Critical Thinking** What can you read directly from the expression $43 + 24h$ that you cannot read directly from the equivalent expression $25 + 10h + 18 + 14h$?

_____

YOUR TURN

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Simplify each expression.

2. $\left(3x + \frac{1}{2}\right) + \left(7x - 4\frac{1}{2}\right)$ 3. $(-0.25x - 3) - (1.5x + 1.4)$

_____   _____

Using the Distributive Property

You can use the Distributive Property to remove the parentheses from an algebraic expression like $3(x + 5)$. Sometimes this is called "simplifying" or "expanding" the expression. Multiply the quantity in front of parentheses by each term within parentheses: $3(x + 5) = 3 \cdot x + 3 \cdot 5 = 3x + 15$.

EXAMPLE 2 🌐 Real World

FL 7.EE.1.1, 7.EE.1.2

Marc is selling tickets for a high school band concert. The band gets to keep 25% of the money he collects from ticket sales to put toward new uniforms. Write an expression to represent how much the band gets to keep.

Let a represent the number of adult tickets he sells.

Let y represent the number of youth tickets he sells.

ADMIT ONE | **Northfield High School** | **Fall Band Concert** | Wednesday, Nov. 12 7:00 P.M. Adults **$16.60** Youth **$12.20** | **TICKET**

The expression $16.60a + 12.20y$ represents the amount of money Marc collects from ticket sales.

Write 25% as a decimal: 0.25

Write an expression to represent 25% of the money he collects:

0.25	×	(16.60a	+	12.20y)
25%	of	adult ticket sales	and	youth ticket sales

Use the Distributive Property to simplify the expression.

$0.25(16.60a) + 0.25(12.20y) = 4.15a + 3.05y$

Reflect

4. **Analyze Relationships** Instead of using the Distributive Property to expand $0.25 \times (16.60a + 12.20y)$, could you have first found the sum $16.60a + 12.20y$? Explain.

_____

_____

YOUR TURN ⬇

Simplify each expression.

5. $7(9k + 6m)$

6. $0.2(3b - 15c)$

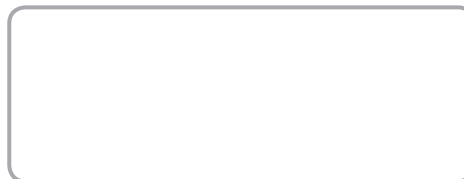7. $\frac{2}{3}(6e + 9f - 21g)$

_____ _____ _____

Factoring Expressions

A factor is a number that is multiplied by another number to get a product. To **factor** is to write a number or an algebraic expression as a product.

Factor $4x + 8$.
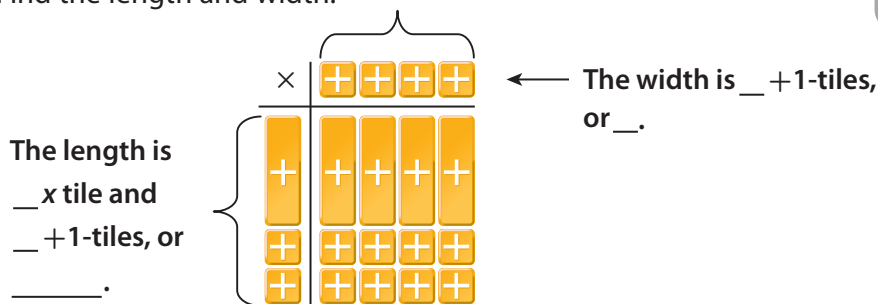
A Model the expression with algebra tiles.

Use _____ positive x tiles and _____ +1-tiles.

B Arrange the tiles to form a rectangle. The total area represents $4x + 8$.

C Since the length multiplied by the width equals the area, the length and the width of the rectangle are the factors of $4x + 8$. Find the length and width.

The length is __ x tile and __ +1-tiles, or _____.

$\times$ | [+][+][+][+]   ←   The width is __ +1-tiles, or __.

D Use the expressions for the length and width of the rectangle

to write the area of the rectangle, $4x + 8$, in factored form. _____

Reflect

8. **Communicate Mathematical Ideas** How could you use the Distributive Property to check your factoring?

_____

YOUR TURN

Factor each expression.

9. $2x + 2$ **10.** $3x + 9$ **11.** $5x + 15$ **12.** $4x + 16$

_____   _____   _____   _____

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1. The manager of a summer camp has 14 baseballs and 23 tennis balls. The manager buys some boxes of baseballs with 12 baseballs to a box and an equal number of boxes of tennis balls with 16 tennis balls to a box. Write an expression to represent the total number of balls. (Example 1)

STEP 1 Write expressions for the total number of baseballs and tennis balls. Let n represent the number of boxes of each type.

baseballs: _____ + (_____)n tennis balls: _____ + (_____)n

STEP 2 Find an expression for the total number of balls.

_____ + _____ + _____ + _____   *Combine the two expressions.*

_____ + _____ + _____ + _____   *Use the Commutative Property.*

_____ + _____   *Combine like terms.*

So, the total number of baseballs and tennis balls is _____ + _____.

2. Use the expression you found above to find the total number of baseballs and tennis balls if the manager bought 9 boxes of each type. (Example 1) _____

Use the Distributive Property to expand each expression. (Example 2)

3. $0.5(12m - 22n)$

$0.5(12m - 22n) = 0.5($_____$) - 0.5($_____$)$ *Distribute 0.5 to both terms in parentheses.*

$=$ _____ $-$ _____   *Multiply.*

4. $\frac{2}{3}(18x + 6z)$

$\frac{2}{3}($_____$) + \frac{2}{3}($_____$) =$ _____ $+$ _____

Factor each expression. (Example 3)

5. $2x + 12$ **6.** $12x + 24$ **7.** $7x + 35$

_____   _____   _____

? ESSENTIAL QUESTION CHECK-IN

8. What is the relationship between multiplying and factoring?

_____

_____

_____

6.1 Independent Practice

FL 7.EE.1.1, 7.EE.1.2

Write and simplify an expression for each situation.

9. A company rents out 15 food booths and 20 game booths at the county fair. The fee for a food booth is $100 plus $5 per day. The fee for a game booth is $50 plus $7 per day. The fair lasts for d days, and all the booths are rented for the entire time. Write and simplify an expression for the amount in dollars that the company is paid.
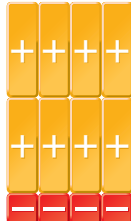
_____

10. A rug maker is using a pattern that is a rectangle with a length of 96 inches and a width of 60 inches. The rug maker wants to increase each dimension by a different amount. Let ℓ and w be the increases in inches of the length and width. Write and simplify an expression for the perimeter of the new pattern.
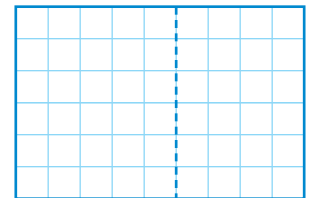
_____

In 11–12, identify the two factors that were multiplied together to form the array of tiles. Then identify the product of the two factors.
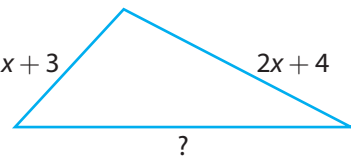
11. _____

12. _____

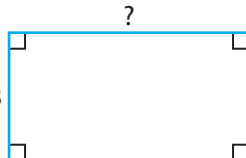**13.** Explain how the figure illustrates that $6(9) = 6(5) + 6(4)$.

_____

_____

_____

_____

In 14–15, the perimeter of the figure is given. Find the length of the indicated side.

14.

$x + 3$ $2x + 4$

?

Perimeter $= 6x$ _____

15.

?

$3x - 3$

Perimeter $= 10x + 6$ _____

16. Persevere in Problem Solving The figures show the dimensions of a tennis court and a basketball court given in terms of the width x in feet of the tennis court.

Tennis x

$2x + 6$

Basketball $\frac{1}{2}x + 32$

$3x - 14$

a. Write an expression for the perimeter of each court. _____

b. Write an expression that describes how much greater the perimeter of the basketball court is than the perimeter of the tennis court. _____

c. Suppose the tennis court is 36 feet wide. Find all dimensions of the two courts. _____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

17. Draw Conclusions Use the figure to find the product $(x + 3)(x + 2)$. (*Hint*: Find the area of each small square or rectangle, then add.)

x 1 1 1

x

1

1
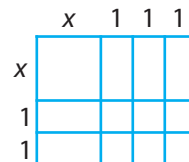
$(x + 3)(x + 2) =$ _____

18. Communicate Mathematical Ideas Desmond claims that the product shown at the right illustrates the Distributive Property. Do you agree? Explain why or why not.

$$\begin{array}{r} 58 \\ \times\ 23 \\ \hline 174 \\ 1160 \\ \hline 1{,}334 \end{array}$$

_____

_____

_____

19. Justify Reasoning Describe two different ways that you could find the product 8×997 using mental math. Find the product and explain why your methods work.

_____

_____

_____

_____

One-Step Equations with Rational Coefficients

? ESSENTIAL QUESTION

How do you use one-step equations with rational coefficients to solve problems?

One-Step Equations

You have written and solved one-step equations involving whole numbers. Now you will learn to work with equations containing negative numbers.

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EXAMPLE 1   Real World

FL 7.EE.2.4

Use inverse operations to solve each equation.

A $x + 3.2 = -8.5$

$$x + 3.2 = -8.5$$
$$\underline{-3.2 \quad\quad -3.2}$$ Subtract 3.2 from both sides.
$$x = -11.7$$

B $-\frac{2}{3} + y = 8$

$$-\frac{2}{3} + y = 8$$
$$\underline{+\frac{2}{3} \quad\quad +\frac{2}{3}}$$ Add $\frac{2}{3}$ to both sides.
$$y = 8\frac{2}{3}$$

$-7.5 = -1.5n$

C $30 = -0.5a$

$$\frac{30}{-0.5} = \frac{-0.5a}{-0.5}$$ Divide both sides by -0.5.
$$-60 = a$$

D $-\frac{q}{3.5} = 9.2$

$$-\frac{q}{3.5}(-3.5) = 9.2(-3.5)$$ Multiply both sides by -3.5.
$$q = -32.2$$
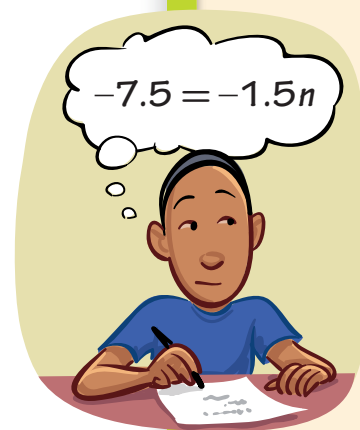
YOUR TURN

Use inverse operations to solve each equation.

1. $4.9 + z = -9$ **2.** $r - 17.1 = -4.8$ **3.** $-3c = 36$

_____   _____   _____

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# Writing and Solving One-Step Addition and Subtraction Equations

Negative numbers often appear in real-world situations. For example, elevations below sea level are represented by negative numbers. When you increase your elevation, you are moving in a positive direction. When you decrease your elevation, you are moving in a negative direction.

EXAMPLE 2   Real World   **FL** **7.EE.2.4**

A scuba diver is exploring at an elevation of −12.2 meters. As the diver rises to the surface, she plans to stop and rest briefly at a reef that has an elevation of −4.55 meters. Find the vertical distance that the diver will travel.

— 0 m

— −4.55 m

— −12.2 m
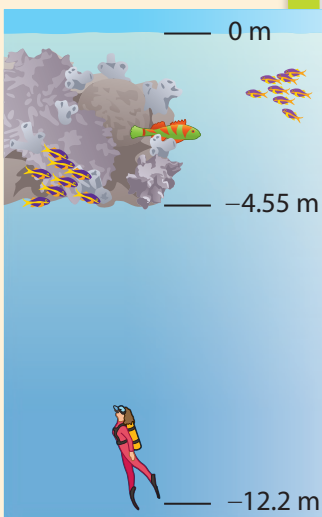
STEP 1   Write an equation. Let x represent the vertical distance between her initial elevation and the elevation of the reef.

$$-12.2 + x = -4.55$$

STEP 2   Solve the equation using an inverse operation.

$$\begin{aligned} -12.2 + x &= -4.55 \\ +12.2 \qquad\quad &+12.2 \end{aligned} \qquad \text{Add 12.2 to both sides.}$$
$$x = 7.65$$

The diver will travel a vertical distance of 7.65 meters.

Reflect

4.   Make a Prediction Explain how you know whether the diver is moving in a positive or a negative direction before you solve the equation.

_____

_____

_____

YOUR TURN

5.   An airplane descends 1.5 miles to an elevation of 5.25 miles. Find the elevation of the plane before its descent.

_____

_____

Writing and Solving One-Step Multiplication and Division Problems

Temperatures can be both positive and negative, and they can increase or decrease during a given period of time. A decrease in temperature is represented by a negative number. An increase in temperature is represented by a positive number.

EXAMPLE 3 — Real World — FL 7.EE.2.4

Between the hours of 10 P.M. and 6 A.M., the temperature decreases an average of $\frac{3}{4}$ of a degree per hour. How many minutes will it take for the temperature to decrease by 5 °F?

STEP 1 Write an equation. Let x represent the number of hours it takes for the temperature to decrease by 5 °F.

$$-\frac{3}{4}x = -5$$

STEP 2 Solve the equation using an inverse operation.

$$-\frac{3}{4}x = -5$$

$$-\frac{4}{3}\left(-\frac{3}{4}x\right) = -\frac{4}{3}(-5) \qquad \text{Multiply both sides by } -\frac{4}{3}.$$

$$x = \frac{20}{3}$$

STEP 3 Convert the number of hours to minutes.

$$\frac{20}{3} \text{ hours} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = 400 \text{ minutes}$$

It takes 400 minutes for the temperature to decrease by 5 °F.

Math Talk
Mathematical Practices

Why is multiplying by $-\frac{4}{3}$ the inverse of multiplying by $-\frac{3}{4}$?

YOUR TURN

6. The value of a share of stock decreases in value at a rate of $1.20 per hour during the first 3.5 hours of trading. Write and solve an equation to find the decrease in the value of the share of stock during that time.

_____

7. After a power failure, the temperature in a freezer increased at an average rate of 2.5 °F per hour. The total increase was 7.5 °F. Write and solve an equation to find the number of hours until the power was restored.

_____

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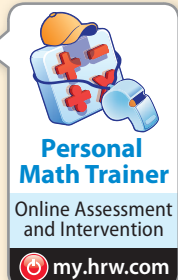
The table shows the average temperature in Barrow, Alaska, for three months during one year.

Month	Average Temperature (°F)
January	−13.4
June	34.0
November	−1.7

1. How many degrees warmer is the average temperature in November than in January? (Examples 1 and 2)

STEP 1 Write an equation. Let x represent _____

_____.

$x +$ _____ $=$ _____ , or $x -$ _____ $=$ _____

STEP 2 Solve the equation. Show your work.

The average temperature in November

is _____ warmer.

2. Suppose that during one period of extreme cold, the average daily temperature decreased $1\frac{1}{2}$ °F each day. How many days did it take for the temperature to decrease by 9 °F? (Examples 1 and 3)

STEP 1 Write an equation. Let x represent _____

_____.

_____ $x =$ _____

STEP 2 Solve the equation. Show your work.

It took _____ days for the temperature to decrease by 9 °F.

Use inverse operations to solve each equation. (Example 1)

3. $-2x = 34$

4. $y - 3.5 = -2.1$

5. $\frac{2}{3}z = -6$

_____   _____   _____

? ESSENTIAL QUESTION CHECK-IN

6. How does writing an equation help you solve a problem?

_____

_____

6.2 Independent Practice

FL 7.EE.2.4

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The table shows the elevation in feet at the peaks of several mountains. Use the table for 7–9.

Mountain	Elevation (feet)
Mt. McKinley	20,321.5
K2	28,251.31
Tupungato	22,309.71
Dom	14,911.42

7. Mt. Everest is 8,707.37 feet higher than Mt. McKinley. What is the elevation of Mt. Everest?

_____

8. Liam descended from the summit of K2 to an elevation of 23,201.06 feet. How many feet did Liam descend? What was his change in elevation?

_____

9. K2 is 11,194.21 feet higher than Mt. Kenya. Write and solve an equation to find the elevation of Mt. Kenya.

_____

_____

10. A hot air balloon begins its descent at a rate of $22\frac{1}{2}$ feet per minute. How long will it take for the balloon's elevation to change by −315 feet?

_____

11. During another part of its flight, the balloon in Exercise 10 had a change in elevation of −901 feet. What was its rate of descent?

_____

The table shows the average temperatures in several states from January through March. Use the table for 12–14.

State	Average Temperature (°C)
Florida	18.1
Minnesota	−2.5
Montana	−0.7
Texas	12.5

12. Write and solve an equation to find how much warmer Montana's average 3-month temperature is than Minnesota's.

_____

13. How much warmer is Florida's average 3-month temperature than Montana's?

_____

14. How would the average temperature in Texas have to change to match the average temperature in Florida?

_____

15. A football team has a net yardage of $-26\frac{1}{3}$ yards on a series of plays. The team needs a net yardage of 10 yards to get a first down. How many yards do they have to get on their next play to get a first down?

_____

16. A diver begins at sea level and descends vertically at a rate of $2\frac{1}{2}$ feet per second. How long does the diver take to reach

−15.6 feet? _____

17. **Analyze Relationships** In Exercise 16, what is the relationship between the rate at which the diver descends, the elevation he reaches, and the time it takes to reach that elevation?

_____

18. **Check for Reasonableness** Jane withdrew money from her savings account in each of 5 months. The average amount she withdrew per month was $45.50. How much did she withdraw in all during the 5 months? Show that your answer is reasonable.

_____

_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

Work Area

19. **Justify Reasoning** Consider the two problems below. Which values in the problems are represented by negative numbers? Explain why.

(1) A diver below sea level ascends 25 feet to a reef at −35.5 feet. What was the elevation of the diver before she ascended to the reef?

(2) A plane descends 1.5 miles to an elevation of 3.75 miles. What was the elevation of the plane before its descent?

_____

_____

_____

20. **Analyze Relationships** How is solving $-4x = -4.8$ different from solving $-\frac{1}{4}x = -4.8$? How are the solutions related?

_____

_____

_____

21. **Communicate Mathematical Ideas** Flynn opens a savings account. In one 3-month period, he makes deposits of $75.50 and $55.25. He makes withdrawals of $25.15 and $18.65. His balance at the end of the 3-month period is $210.85. Explain how you can find his initial deposit amount.

_____

_____

_____

Writing Two-Step Equations

? ESSENTIAL QUESTION

How do you write a two-step equation?

EXPLORE ACTIVITY   FL   **Prep for 7.EE.2.4**

Modeling Two-Step Equations

You can use algebra tiles to model two-step equations.

KEY

$+$ = positive variable

$-$ = negative variable

$+$ = 1 $-$ = −1

Use algebra tiles to model $3x - 4 = 5$.

A How can you model the left side of the equation?

_____

B How can you model the right side of the equation?

_____

C Use algebra tiles or draw them to model the equation on the mat.

Math Talk
Mathematical Practices

Why is the mat divided into two equal halves with a line?

Reflect

1. **What If?** How would you change the equation in the Explore Activity to model $-3x + 4 = 5$?

_____

_____

_____

Writing Two-Step Equations

You can write two-step equations to represent real-world problems by translating the words of the problems into numbers, variables, and operations.

EXAMPLE 1 🌐 Real World   🏴 **FL** **7.EE.2.4**

A one-year membership to Metro Gym costs $460. There is a fee of $40 when you join, and the rest is paid monthly. Write an equation to represent what you will pay monthly for the yearlong membership. Write an equation that would help members find how much they pay per month.

STEP 1 Identify what you are trying to find. This will be the variable in the equation.

Let m represent the amount of money members pay per month.

STEP 2 Identify important information in the problem that can be used to help write an equation.

one-time joining fee: **$40**
fee charged for 1 year: **$12 \cdot m$**
total cost for the year: **$460**

> Convert 1 year into 12 months to find how much members pay per month.

STEP 3 Use words in the problem to tie the information together and write an equation.

One-time joining fee	plus	12	times	monthly cost	equals	$460
↓	↓	↓	↓	↓	↓	↓
$40	+	12	·	m	=	$460

The equation $40 + 12m = 460$ can help members find out their monthly fee.

Reflect

2. Multiple Representations Why would this equation for finding the monthly fee be difficult to model with algebra tiles?

_____

3. Can you rewrite the equation in the form $52m = 460$? Explain.

_____

_____

4. Billy has a gift card with a $150 balance. He buys several video games that cost $35 each. After the purchases, his gift card balance is $45. Write an equation to help find out how many video games Billy bought.

Writing a Verbal Description of a Two-Step Equation

You can also write a verbal description to fit a two-step equation.

EXAMPLE 2   Real World   FL   7.EE.2.4

Write a corresponding real-world problem to represent $5x + 50 = 120$.

STEP 1 Analyze what each part of the equation means mathematically.

x is the solution of the problem, the quantity you are looking for.

$5x$ means that, for a reason given in the problem, the quantity you are looking for is multiplied by 5.

$+ 50$ means that, for a reason given in the problem, 50 is added to $5x$.

$= 120$ means that after multiplying the solution x by 5 and adding 50 to it, the result is 120.

STEP 2 Think of some different situations in which a quantity x might be multiplied by 5.

You have x number of books, each weighing 5 pounds, and you want to know their total weight.	You save $5 each week for x weeks and want to know the total amount you have saved.

STEP 3 Build on the situation and adjust it to create a verbal description that takes all of the information of the equation into account.

- A publisher ships a package of x number of books each weighing 5 pounds, plus a second package weighing 50 pounds. The total weight of both packages is 120 pounds. How many books are being shipped?

- Leon receives a birthday gift of $50 from his parents and decides to save it. Each week he adds $5 to his savings. How many weeks will it take for him to save $120?

My Notes
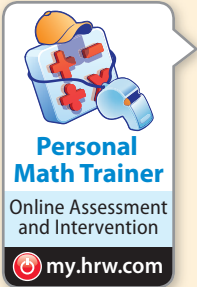
YOUR TURN

5. Write a real-world problem that can be represented by $10x + 40 = 100$.

_____

_____

Guided Practice

Draw algebra tiles to model the given two-step equation. (Explore Activity)

1. $2x + 5 = 7$

2. $-3 = 5 - 4x$

3. A group of adults plus one child attend a movie at Cineplex 15. Tickets cost $9 for adults and $6 for children. The total cost for the movie is $78. Write an equation to find the number of adults in the group. (Example 1) _____

4. Break down the equation $2x + 10 = 16$ to analyze each part. (Example 2)

   x is _____ of the problem.

   2x is the quantity you are looking for _____.

   + 10 means 10 is _____. **= 16** means the _____ is 16.

5. Write a corresponding real-world problem to represent $2x - 125 = 400$.

   (Example 2) _____

   _____

? ESSENTIAL QUESTION CHECK-IN

6. Describe the steps you would follow to write a two-step equation you can use to solve a real-world problem.

   _____

   _____

   _____

6.3 Independent Practice

FL 7.EE.2.4

7. Describe how to model $-3x + 7 = 28$ with algebra tiles.

_____

_____

_____

8. Val rented a bicycle while she was on vacation. She paid a flat rental fee of $55.00, plus $8.50 each day. The total cost was $123. Write an equation you can use to find the number of days she rented the bicycle.

_____

9. A restaurant sells a coffee refill mug for $6.75. Each refill costs $1.25. Last month Keith spent $31.75 on a mug and refills. Write an equation you can use to find the number of refills that Keith bought.

_____

10. A gym holds one 60-minute exercise class on Saturdays and several 45-minute classes during the week. Last week all of the classes lasted a total of 285 minutes. Write an equation you can use to find the number of weekday classes.

_____

11. Multiple Representations There are 172 South American animals in the Springdale Zoo. That is 45 more than half the number of African animals in the zoo. Write an equation you could use to find n, the number of African animals in the zoo.

_____

12. A school bought $548 in basketball equipment and uniforms costing $29.50 each. The total cost was $2,023. Write an equation you can use to find the number of uniforms the school purchased.

_____

13. Financial Literacy Heather has $500 in her savings account. She withdraws $20 per week for gas. Write an equation Heather can use to see how many weeks it will take her to have a balance of $220.

_____

14. Critique Reasoning For $9x + 25 = 88$, Deena wrote the situation "I bought some shirts at the store for $9 each and received a $25 discount. My total bill was $88. How many shirts did I buy?"

a. What mistake did Deena make?

_____

_____

_____

b. Rewrite the equation to match Deena's situation.

_____

c. How could you rewrite the situation to make it fit the equation?

_____

_____

_____

_____

_____

15. Multistep Sandy charges each family that she babysits a flat fee of $10 for the night and an extra $5 per child. Kimmi charges $25 per night, no matter how many children a family has.

a. Write a two-step equation that would compare what the two girls

charge and find when their fees are the same. _____

b. How many children must a family have for Sandy and Kimmi to

charge the same amount? _____

c. The Sanderson family has five children. Which babysitter should they choose if they wish to save some money on babysitting, and why?

_____

_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

16. Analyze Relationships Each student wrote a two-step equation. Peter wrote the equation $4x - 2 = 10$, and Andres wrote the equation $16x - 8 = 40$. The teacher looked at their equations and asked them to compare them. Describe one way in which the equations are similar.

_____

_____

_____

_____

17. What's the Error? Damon has 5 dimes and some nickels in his pocket, worth a total of $1.20. To find the number of nickels Damon has, a student wrote the equation $5n + 50 = 1.20$. Find the error in the student's equation.

_____

_____

_____

18. Represent Real-World Problems Write a real-world problem you could answer by solving the equation $-8x + 60 = 28$.

_____

_____

_____

_____

Work Area

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Solving Two-Step Equations

? ESSENTIAL QUESTION

How do you solve a two-step equation?

Modeling and Solving Two-Step Equations

You can solve two-step equations using algebra tiles.

Math On the Spot
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EXAMPLE 1
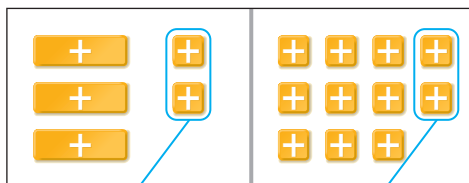FL   7.EE.2.4

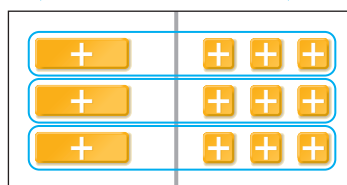**Use algebra tiles to model and solve $3n + 2 = 11$.**

STEP 1 Model the equation.

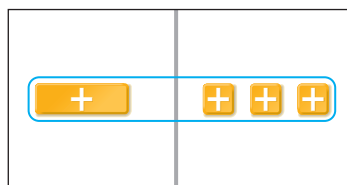**STEP 2** Remove 2 +1-tiles from each side of the mat.

Since there are +1-tiles on both sides of the equation, you can remove, or subtract, 2 +1-tiles from each side to help isolate the variable.

STEP 3 Divide each side into 3 equal groups.

STEP 4 The solution is $n = 3$.

YOUR TURN

Use algebra tiles to model and solve each equation.

1. $2x + 5 = 11$ _____

2. $3n - 1 = 8$ _____

3. $2a - 3 = -5$ _____

4. $-4y + 2 = -2$ _____

Personal Math Trainer

Online Assessment and Intervention

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Solving Two-Step Equations

You can use inverse operations to solve equations with more than one operation.

EXAMPLE 2   Real World   FL   7.EE.2.4a

A dog sled driver added more gear to the sled, doubling its weight. This felt too heavy, so the driver removed 20 pounds to reach the final weight of 180 pounds. Write and solve an equation to find the sled's original weight.

STEP 1   Write an equation. Let w represent the original weight of the sled.

$2w - 20 = 180$.

STEP 2   Solve the equation.

$$2w - 20 = 180$$
$$\underline{+\,20 \quad +\,20} \qquad \text{Add 20 to both sides.}$$
$$2w \quad\quad = 200$$
$$\frac{2w}{2} = \frac{200}{2} \qquad \text{Divide both sides by 2.}$$
$$w = 100$$

The sled's original weight was 100 pounds.

Reflect

5. **Analyze Relationships** Describe how you could find the original weight of the sled using only arithmetic. Compare this method with the method shown in Example 2.

_____

_____

_____

_____

YOUR TURN

Solve each problem by writing and solving an equation.

6. The Wilsons have triplets and another child who is ten years old. The sum of the ages of their children is 37. How old are the triplets?

_____

7. Five less than the quotient of a number and 4 is 15. What is the number?

_____

Two-Step Equations with Negative Numbers

Many real-world quantities such as altitude or temperature involve negative numbers. You solve equations with negative numbers just as you did equations with positive numbers.

EXAMPLE 3 · Real World

FL 7.EE.2.4a

A To convert a temperature from degrees Fahrenheit to degrees Celsius, first subtract 32. Then multiply the result by $\frac{5}{9}$. An outdoor thermometer showed a temperature of $-10\,°C$. What was the temperature in degrees Fahrenheit?

STEP 1 Write an equation. Let x represent the temperature in degrees Fahrenheit.

$$-10 = \frac{5}{9}(x - 32)$$

STEP 2 Solve the equation.

$$\frac{9}{5}(-10) = \frac{9}{5}\left(\frac{5}{9}(x-32)\right) \quad \text{Multiply both sides by } \frac{9}{5}.$$

$$-18 = x - 32$$

$$\underline{+32 \qquad +32} \qquad \text{Add 32 to both sides.}$$

$$14 = x$$

The temperature was 14 degrees Fahrenheit.

Math Talk
Mathematical Practices
How can you check the solution?

B An airplane flies at an altitude of 38,000 feet. As it nears the airport, the plane begins to descend at a rate of 600 feet per minute. At this rate, how many minutes will the plane take to descend to 18,800 feet?

STEP 1 Write an equation. Let m represent the number of minutes.

$$38{,}000 - 600m = 18{,}800$$

STEP 2 Solve the equation. Start by isolating the term that contains the variable.

$$38{,}000 - 600m = 18{,}800$$

$$\underline{-38{,}000 \qquad\qquad -38{,}000} \qquad \text{Subtract 38,000 from both sides.}$$

$$-600m = -19{,}200$$

$$\frac{-600m}{-600} = \frac{-19{,}200}{-600} \qquad \text{Divide both sides by } -600.$$

$$m = 32$$

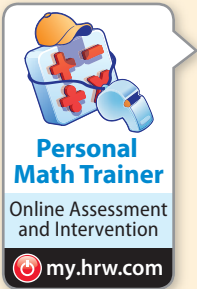The plane will take 32 minutes to descend to 18,800 feet.

YOUR TURN

Solve each problem by writing and solving an equation.

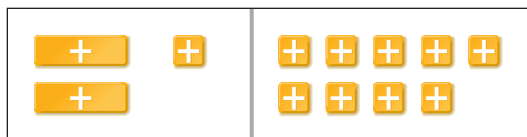**8.** What is the temperature in degrees Fahrenheit of a freezer kept at -20 °C?

_____

9. Jenny earned 92 of a possible 120 points on a test. She lost 4 points for each incorrect answer. How many incorrect answers did she have?

_____

Guided Practice

The equation $2x + 1 = 9$ is modeled below. (Example 1)

1. To solve the equation with algebra tiles, first remove _____ .

Then divide each side into _____.

2. The solution is $x =$ _____ .

Solve each problem by writing and solving an equation.

3. A rectangular picture frame has a perimeter of 58 inches. The height of the frame is 18 inches. What is the width of the frame? (Example 2)

_____

4. A school store has 1200 pencils in stock, and sells an average of 24 pencils per day. The manager reorders when the number of pencils in stock is 500. In how many days will the manager have to reorder? (Example 3)

_____

❓ ESSENTIAL QUESTION CHECK-IN

5. How can you decide which operations to use to solve a two-step equation?

_____

_____

_____

6.4 Independent Practice

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Solve.

6. $9s + 3 = 57$

_____

7. $4d + 6 = 42$

_____

8. $-3y + 12 = -48$

_____

9. $\frac{k}{2} + 9 = 30$

_____

10. $\frac{g}{3} - 7 = 15$

_____

11. $\frac{z}{5} + 3 = -35$

_____

12. $-9h - 15 = 93$

_____

13. $-3(n + 5) = 12$

_____

14. $-17 + \frac{b}{8} = 13$

_____

15. $7(c - 12) = -21$

_____

16. $-3 + \frac{p}{7} = -5$

_____

17. $46 = -6t - 8$

_____

18. After making a deposit, Puja had $264 in her savings account. She noticed that if she added $26 to the amount originally in the account and doubled the sum, she would get the new amount. How much did she originally have in the account?

_____

19. The current temperature in Smalltown is 20 °F. This is 6 degrees less than twice the temperature that it was six hours ago. What was the temperature in Smalltown six hours ago?

_____

20. One reading at an Arctic research station showed that the temperature was −35 °C. What is this temperature in degrees Fahrenheit?

_____

21. Artaud noticed that if he takes the opposite of his age and adds 40, he gets the number 28. How old is Artaud?

_____

22. Sven has 11 more than twice as many customers as when he started selling newspapers. He now has 73 customers. How many did he have when he started?

_____

23. Paula bought a ski jacket on sale for $6 less than half its original price. She paid $88 for the jacket. What was the original price?

_____

24. The McIntosh family went apple picking. They picked a total of 115 apples. The family ate a total of 8 apples each day. After how many days did they have 19 apples left?

_____

Use a calculator to solve each equation.

25. $-5.5x + 0.56 = -1.64$

_____

26. $-4.2x + 31.5 = -65.1$

_____

27. $\frac{k}{5.2} + 81.9 = 47.2$

_____

28. Write a two-step equation that involves multiplication and subtraction, includes a negative coefficient, and has a solution of $x = 7$.

_____

29. Write a two-step equation involving division and addition that has a solution of $x = -25$

_____

30. **Explain the Error** A student's solution to the equation $3x + 2 = 15$ is shown. Describe and correct the error that the student made.

$$3x + 2 = 15 \qquad \text{Divide both sides by 3.}$$

$$x + 2 = 5 \qquad \text{Subtract 2 from both sides.}$$

$$x = 3$$

_____

_____

31. **Multiple Representations** Explain how you could use the work backward problem-solving strategy to solve the equation $\frac{x}{4} - 6 = 2$.

_____

_____

_____

_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

32. **Reason Abstractly** The formula $F = 1.8C + 32$ allows you to find the Fahrenheit (F) temperature for a given Celsius (C) temperature. Solve the equation for C to produce a formula for finding the Celsius temperature for a given Fahrenheit temperature.

_____

33. **Reason Abstractly** The equation $P = 2(\ell + w)$ can be used to find the perimeter P of a rectangle with length ℓ and width w. Solve the equation for w to produce a formula for finding the width of a rectangle given its perimeter and length.

_____

34. **Reason Abstractly** Solve the equation $ax + b = c$ for x.

_____

Ready to Go On?

6.1 Algebraic Expressions

1. The Science Club went on a two-day field trip. The first day the members paid $60 for transportation plus $15 per ticket to the planetarium. The second day they paid $95 for transportation plus $12 per ticket to the geology museum. Write an expression to represent the total cost for two days

for the *n* members of the club. _____

6.2 One-Step Equations with Rational Coefficients

Solve.

2. $h + 9.7 = -9.7$ _____

3. $-\frac{3}{4} + p = \frac{1}{2}$ _____

4. $-15 = -0.2k$ _____

5. $\frac{y}{-3} = \frac{1}{6}$ _____

6. $-\frac{2}{3}m = -12$ _____

7. $2.4 = -\frac{t}{4.5}$ _____

6.3 Writing Two-Step Equations

8. Jerry started doing sit-ups every day. The first day he did 15 sit-ups. Every day after that he did 2 more sit-ups than he had done the previous day. Today Jerry did 33 sit-ups. Write an equation that could be solved to find the number of days Jerry has been doing sit-ups, not counting the first day.

_____

6.4 Solving Two-Step Equations

Solve.

9. $5n + 8 = 43$ _____

10. $\frac{y}{6} - 7 = 4$ _____

11. $8w - 15 = 57$ _____

12. $\frac{g}{3} + 11 = 25$ _____

13. $\frac{f}{5} - 22 = -25$ _____

14. $-4p + 19 = 11$ _____

? ESSENTIAL QUESTION

15. How can you use two-step equations to represent and solve real-world problems?

_____

_____

Assessment Readiness

Selected Response

1. A taxi cab costs $1.50 for the first mile and $0.75 for each additional mile. Which equation could be solved to find how many miles you can travel in a taxi for $10, given that x is the number of additional miles?

(A) $1.5x + 0.75 = 10$

(B) $0.75x + 1.5 = 10$

(C) $1.5x - 0.75 = 10$

(D) $0.75x - 1.5 = 10$

2. Which is the solution of $\frac{t}{2.5} = -5.2$?

(A) $t = -13$

(B) $t = -2.08$

(C) $t = 2.08$

(D) $t = 13$

3. Which expression is equivalent to $5x - 30$?

(A) $5(x - 30)$

(B) $5(x - 6)$

(C) $5x(x - 6)$

(D) $x(5 - 30)$

4. In a science experiment, the temperature of a substance is changed from 42 °F to -54 °F at an average rate of -12 degrees per hour. Over how many hours does the change take place?

(A) -8 hours

(B) $\frac{1}{8}$ hour

(C) 1 hour

(D) 8 hours

5. Which statement best represents the distance on a number line between -14 and -5?

(A) $-14 - (-5)$

(B) $-14 + (-5)$

(C) $-5 - (-14)$

(D) $-5 + (-14)$

6. Which cereal costs the most per ounce?

(A) $4.92 for 12 ounces

(B) $4.25 for 10 ounces

(C) $5.04 for 14 ounces

(D) $3.92 for 8 ounces

Mini-Task

7. Casey bought 9 tickets to a concert. The total charge was $104, including a $5 service charge.

a. Write an equation you can solve to find c, the cost of one ticket.

_____

b. Explain how you could estimate the solution of your equation.

_____

_____

_____

_____

c. Solve the equation. How much did each ticket cost?

_____

Inequalities

? ESSENTIAL QUESTION

How can you use inequalities to solve real-world problems?

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Real-World Video

Many school groups and other organizations hold events to raise money. Members can write and solve inequalities to represent the financial goals they are trying to achieve.

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Animated Math

Interactively explore key concepts to see how math works.

Personal Math Trainer

Get immediate feedback and help as you work through practice sets.

Are YOU Ready?

Complete these exercises to review skills you will need for this module.

Inverse Operations

EXAMPLE

$3x = 24$ x is multiplied by 3.

$\dfrac{3x}{3} = \dfrac{24}{3}$ Use the inverse operation, division.

    Divide both sides by 3.

$x = 8$

$z + 6 = 4$ 6 is added to z.

$\underline{-6 = -6}$ Use the inverse operation, subtraction.

$z = -2$ Subtract 6 from both sides.

Solve each equation, using inverse operations.

1. $9w = -54$ _____ **2.** $b - 12 = 3$ _____ **3.** $\dfrac{n}{4} = -11$ _____

Locate Points on a Number Line

EXAMPLE
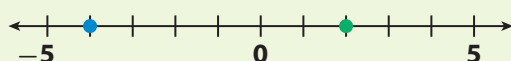
Graph $+2$ by starting at 0 and counting 2 units to the *right*.

Graph -4 by starting at 0 and counting 4 units to the *left*.

Graph each number on the number line.
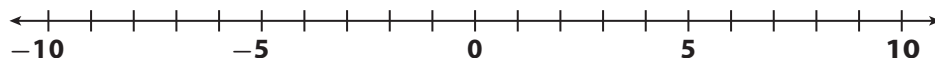
4. 3   **5.** -9 **6.** 7   **7.** -3

Integer Operations

EXAMPLE

$-7 - (-4) = -7 + 4$ To subtract an integer, add its opposite.

$= |-7| - |4|$ The signs are different, so find the difference of the absolute values.

$= 7 - 4$, or 3

$= -3$ Use the sign of the number with the greater absolute value.

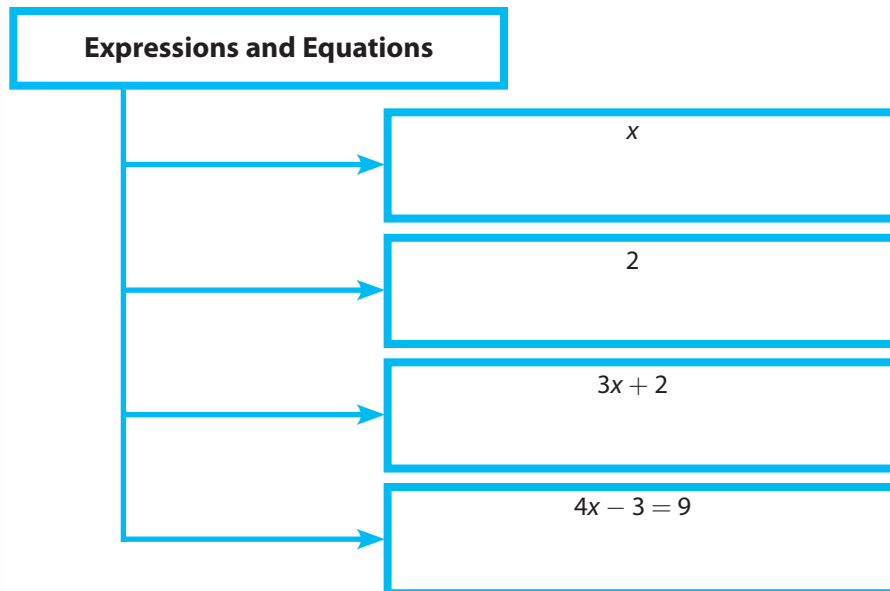**8.** $3 - (-5)$ _____ **9.** $-4 - 5$ _____ **10.** $6 - 10$ _____ **11.** $-5 - (-3)$ _____

12. $8 - (-8)$ _____ **13.** $9 - 5$ _____ **14.** $-3 - 9$ _____ **15.** $0 - (-6)$ _____

Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic. You may put more than one word in each box.

```
┌─────────────────────────────────┐
│  Expressions and Equations      │
└─────────────────────────────────┘
   │
   │      ┌─────────────────────────────────┐
   ├─────▶│              x                  │
   │      └─────────────────────────────────┘
   │
   │      ┌─────────────────────────────────┐
   ├─────▶│              2                  │
   │      └─────────────────────────────────┘
   │
   │      ┌─────────────────────────────────┐
   ├─────▶│            3x + 2               │
   │      └─────────────────────────────────┘
   │
   │      ┌─────────────────────────────────┐
   └─────▶│           4x − 3 = 9            │
          └─────────────────────────────────┘
```

Understand Vocabulary

Complete each sentence, using the review words.

1. A value of the variable that makes the equation true is a _____.

2. The set of all whole numbers and their opposites are _____.

3. An _____ is an expression that contains at least one variable.

Active Reading

Layered Book Before beginning the module, create a layered book to help you learn the concepts in this module. At the top of the first flap, write the title of the module, "Inequalities." Then label each flap with one of the lesson titles in this module. As you study each lesson, write important ideas, such as vocabulary and processes, under the appropriate flap.

Vocabulary

Review Words
✔ algebraic expression *(expresión algebraica)*
  coefficient *(coeficiente)*
✔ constant *(constante)*
✔ equation *(ecuación)*
  greater than *(mayor que)*
✔ inequality *(desigualdad)*
  integers *(enteros)*
  less than *(menor que)*
  operations *(operaciones)*
  solution *(solución)*
✔ variable *(variable)*

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

FL 7.EE.2.4

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Key Vocabulary

inequality *(desigualdad)*
A mathematical sentence that shows that two quantities are not equal.

What It Means to You

You will write an inequality to solve a real-world problem.

UNPACKING EXAMPLE 7.EE.2.4

To rent a certain car for a day costs $39 plus $0.29 for every mile the car is driven. Write an inequality to show the maximum number of miles you can drive and keep the rental cost under $100.

The expression for the cost of the rental is $39 + 0.29m$. The total cost of the rental must be under $100. So the inequality is as shown.

$$39 + 0.29m < 100$$

FL 7.EE.2.4b

Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

Key Vocabulary

solution *(solución)*
The value(s) for the variable that makes the inequality true.

What It Means to You

You will solve inequalities that involve two steps and interpret the solutions.

UNPACKING EXAMPLE 7.EE.2.4b

Solve and graph the solution of $-3x + 7 > -8$.

$-3x + 7 > -8$

$\quad -3x > -7 -8$ Subtract 7 from both sides.

$\quad -3x > -15$ Simplify.

$\quad\quad x < 5$ Divide both sided by -5, and reverse the inequality.

All numbers less than 5 are solutions for this inequality.

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Writing and Solving One-Step Inequalities

? **ESSENTIAL QUESTION**

How do you write and solve one-step inequalities?

EXPLORE ACTIVITY FL **Prep. for 7.EE.2.4b**

Investigating Inequalities

You know that when you perform any of the four basic operations on both sides of an equation, the resulting equation is still true. What effect does performing these operations on both sides of an *inequality* have?

A Complete the table.

Inequality	Add to both sides:	New Inequality	Is new inequality true or false?
$2 \geq -3$	3		
$-1 \leq 6$	-1		
$-8 > -10$	-8		

Reflect

1. Make a Conjecture When you add the same number to both sides of an inequality, is the inequality still true? Explain how you know that your conjecture holds for *subtracting* the same number.

_____

B Complete the table.

Inequality	Divide both sides by:	New Inequality	Is new inequality true or false?
$4 < 8$	4		
$12 \geq -15$	3		
$-16 \leq 12$	-4		
$15 > 5$	-5		

What do you notice when you divide both sides of an inequality by the same negative number?

_____

Reflect

2. **Make a Conjecture** What could you do to make the inequalities that are not true into true statements?

_____

3. **Communicate Mathematical Ideas** Explain how you know that your conjecture holds for multiplying both sides of an inequality by a negative number.

_____

Math On the Spot

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Solving Inequalities Involving Addition and Subtraction

You can use properties of inequality to solve inequalities involving addition and subtraction with rational numbers.

Addition and Subtraction Properties of Inequality	
Addition Property of Inequality	**Subtraction Property of Inequality**
You can add the same number to both sides of an inequality and the inequality will remain true.	You can subtract the same number from both sides of an inequality and the inequality will remain true.

EXAMPLE 1

FL   7.EE.2.4b

Solve each inequality. Graph and check the solution.

A $x + 5 < -12$

STEP 1 Solve the inequality.

$$x + 5 < -12 \qquad \textit{Use the Subtraction Property of Inequality.}$$
$$\underline{-5 \qquad -5} \qquad \textit{Subtract 5 from both sides.}$$
$$x < -17$$
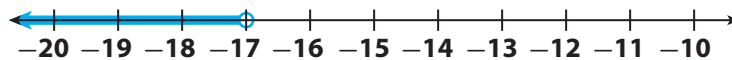
STEP 2 Graph the solution.

−20 −19 −18 −17 −16 −15 −14 −13 −12 −11 −10

STEP 3 Check the solution. Substitute a solution from the shaded part of your number line into the original inequality.

$$-18 + 5 \overset{?}{<} -12 \qquad \textit{Substitute −18 for x into } x + 5 < -12.$$

$$-13 < -12 \qquad \textit{The inequality is true.}$$

Animated Math

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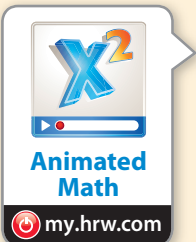
B $8 \leq y - 3$

STEP 1 Solve the inequality.

$$8 \leq y - 3$$
$$\underline{+ 3 \qquad + 3}$$
$$11 \leq y$$

Use the Addition Property of Inequality.

Add 3 to both sides.

You can rewrite $11 \leq y$ as $y \geq 11$.

STEP 2 Graph the solution.
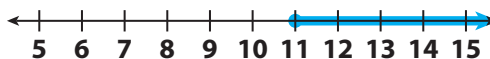
$$5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15$$

STEP 3 Check the solution. Substitute a solution from the shaded part of your number line into the original inequality.

$$8 \overset{?}{\leq} 12 - 3$$

Substitute 12 for y in $8 \leq y - 3$.

$$8 \leq 9$$

The inequality is true.

YOUR TURN

Solve each inequality. Graph and check the solution.

4. $y - 5 \geq -7$

_____

$$-5 \; -4 \; -3 \; -2 \; -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

5. $21 > 12 + x$

_____

$$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$$

Solving Inequalities Involving Multiplication and Division

You can use properties of inequality to solve inequalities involving multiplication and division with rational numbers.

Multiplication and Division Properties of Inequality

- You can multiply or divide both sides of an inequality by the same positive number and the inequality will remain true.

- If you multiply or divide both sides of an inequality by the same negative number, you must reverse the inequality symbol for the statement to still be true.

EXAMPLE 2　　　　　　　　　　　　**FL** 7.EE.2.4b

Solve each inequality. Graph and check the solution.

A $\frac{y}{3} \geq 5$

STEP 1 Solve the inequality.

$3\left(\frac{y}{3}\right) \geq 3(5)$　　**Multiply both sides by 3.**

$y \geq 15$

> Use a closed circle to show that 15 is a solution.

STEP 2 Graph the solution.
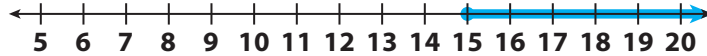
STEP 3 Check the solution by substituting a solution from the shaded part of the graph into the original inequality. For convenience, choose a multiple of 3.

$\frac{18}{3} \overset{?}{\geq} 5$　　　　**Substitute 18 for x in the original inequality.**

$6 \geq 5$　　　**The inequality is true.**

B $-4x > 52$

STEP 1 Solve the inequality.

$-4x > 52$

$\frac{-4x}{-4} < \frac{52}{-4}$　　　**Divide both sides by −4.**
Reverse the inequality symbol.

$x < -13$

STEP 2 Graph the solution.
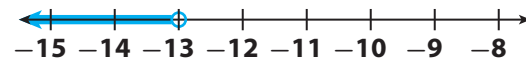
STEP 3 Check your answer using substitution.

$-4(-15) \overset{?}{>} 52$　　**Substitute −15 for x in −4x > 52.**

$60 > 52$　　　**The statement is true.**

YOUR TURN

Solve each inequality. Graph and check the solution.

6. $-10y < 60$ _____

7. $7 \geq -\frac{t}{6}$ _____

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Solving a Real-World Problem

Although elevations below sea level are represented by negative numbers, we often use absolute values to describe these elevations. For example, -50 feet relative to sea level might be described as 50 feet below sea level.

EXAMPLE 3 | Problem Solving | FL 7.EE.2.4, 7.EE.2.4b

A marine submersible descends more than 40 feet below sea level. As it descends from sea level, the change in elevation is -5 feet per second. For how many seconds does it descend?

Analyze Information

Rewrite the question as a statement.

- Find the number of seconds that the submersible descends below sea level.

List the important information:

- Final elevation > 40 feet below sea level or final elevation < -40 feet
- Rate of descent $= -5$ feet per second

Formulate a Plan

Write and solve an inequality. Use this fact:

Rate of change in elevation $\times$ Time in seconds $=$ Final elevation

Solve

$-5t < -40$ *Rate of change $\times$ Time $<$ Final elevation*

$\dfrac{-5t}{-5} > \dfrac{-40}{-5}$ *Divide both sides by -5. Reverse the inequality symbol.*

$t > 8$

The submersible descends for more than 8 seconds.

Justify and Evaluate

Check your answer by substituting a value greater than 8 seconds in the original inequality.

$-5(9) \overset{?}{<} -40$ *Substitute 9 for t in the inequality $-5t < -40$.*

$-45 < -40$ *The statement is true.*

YOUR TURN

8. Every month, \$35 is withdrawn from Tony's savings account to pay for his gym membership. He has enough savings to withdraw no more than \$315. For how many months can Tony pay for his gym membership?

_____

_____

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Guided Practice

Write the resulting inequality. (Explore Activity)
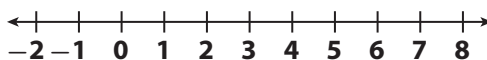
1. $-5 \leq -2$; Add 7 to both sides _____

2. $-6 < -3$; Divide both sides by -3 _____

3. $7 > -4$; Subtract 7 from both sides _____

4. $-1 \geq -8$; Multiply both sides by -2 _____

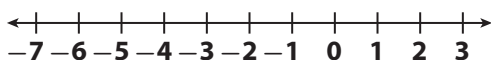**Solve each inequality. Graph and check the solution.** (Examples 1 and 2)
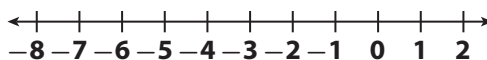
5. $n - 5 \geq -2$ _____

$$\xleftarrow{\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;} \atop{-5\;-4\;-3\;-2\;-1\;\;0\;\;1\;\;2\;\;3\;\;4\;\;5}$$

6. $3 + x < 7$ _____

$$\xleftarrow{\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;} \atop{-2\;-1\;\;0\;\;1\;\;2\;\;3\;\;4\;\;5\;\;6\;\;7\;\;8}$$

7. $-7y \leq 14$ _____

$$\xleftarrow{\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;} \atop{-7\;-6\;-5\;-4\;-3\;-2\;-1\;\;0\;\;1\;\;2\;\;3}$$

8. $\frac{b}{5} > -1$ _____

$$\xleftarrow{\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;|\;\;} \atop{-8\;-7\;-6\;-5\;-4\;-3\;-2\;-1\;\;0\;\;1\;\;2}$$
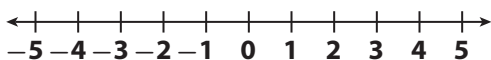
9. For a scientific experiment, a physicist must make sure that the temperature of a metal at 0 °C gets no colder than −80 °C. The physicist changes the metal's temperature at a steady rate of −4 °C per hour. For how long can the physicist change the temperature? (Example 3)

  a. Let t represent temperature in degrees Celsius. Write an inequality. Use the fact that the rate of change in temperature times the number of hours equals the final temperature.

  _____

  b. Solve the inequality in part **a**. How long can the physicist change the temperature of the metal?

  _____

  c. The physicist has to repeat the experiment if the metal gets cooler than −80 °C. How many hours would the physicist have to cool the metal for this to happen?

  _____

? ESSENTIAL QUESTION CHECK-IN

10. Suppose you are solving an inequality. Under what circumstances do you reverse the inequality symbol?
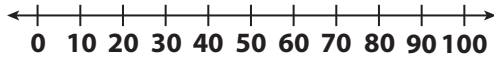
_____

7.1 Independent Practice

FL 7.EE.2.4, 7.EE.2.4b

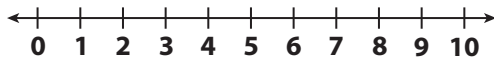**Personal Math Trainer**

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In 11–16, solve each inequality. Graph and check the solution.

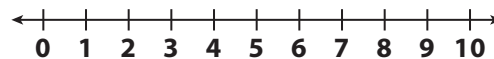**11.** $x - 35 > 15$ _____

```
0  10 20 30 40 50 60 70 80 90 100
```
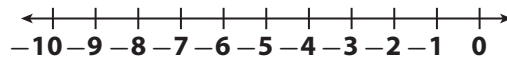
12. $193 + y \geq 201$ _____

```
0  1  2  3  4  5  6  7  8  9  10
```

13. $-\dfrac{q}{7} \geq -1$ _____

```
0  1  2  3  4  5  6  7  8  9  10
```

14. $-12x < 60$ _____

```
-10 -9 -8 -7 -6 -5 -4 -3 -2 -1  0
```

15. $5 > z - 3$ _____

```
0  1  2  3  4  5  6  7  8  9  10
```

16. $0.5 \leq \dfrac{y}{8}$ _____

```
0  1  2  3  4  5  6  7  8  9  10
```
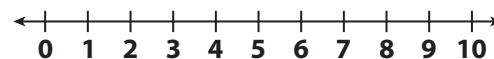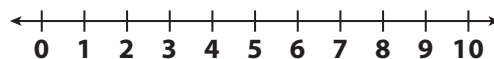
17. The vet says that Lena's puppy will grow to be at most 28 inches tall. Lena's puppy is currently 1 foot tall. How many more inches will the puppy grow?

_____

18. In a litter of 7 kittens, each kitten weighs less than 3.5 ounces. Find all the possible values of the combined weights of the kittens.

_____

19. Geometry The sides of the hexagon shown are equal in length. The perimeter of the hexagon is at most 42 inches. Find the possible side lengths of the hexagon.

_____

20. To get a free meal at his favorite restaurant, Tom needs to spend $50 or more at the restaurant. He has already spent $30.25. How much more does Tom need to spend to get his free meal?

_____

21. To cover a rectangular region of her yard, Penny needs at least 170.5 square feet of sod. The length of the region is 15.5 feet. What are the possible widths of the region?

_____

22. Draw Conclusions A submarine descends from sea level to the entrance of an underwater cave. The elevation of the entrance is -120 feet. The rate of change in the submarine's elevation is no greater than -12 feet per second. Can the submarine reach the entrance to the cave in less than 10 seconds? Explain.

_____

_____

_____

The sign shows some prices at a produce stand.

23. Selena has $10. What is the greatest amount of spinach she can buy?

_____

Produce	Price per Pound
Onions	$1.25
Yellow Squash	$0.99
Spinach	$3.00
Potatoes	$0.50

24. Gary has enough money to buy at most 5.5 pounds of potatoes. How much money does Gary have?

_____

25. Florence wants to spend no more than $3 on onions. Will she be able to buy 2.5 pounds of onions? Explain.

_____

_____

H.O.T. **FOCUS ON HIGHER ORDER THINKING**

Work Area

26. **Counterexamples** John says that if one side of an inequality is 0, you don't have to reverse the inequality symbol when you multiply or divide both sides by a negative number. Find an inequality that you can use to disprove John's statement. Explain your thinking.

_____

_____

_____

_____

27. **Look for a Pattern** Solve $x + 1 > 10$, $x + 11 > 20$, and $x + 21 > 30$. Describe a pattern. Then use the pattern to predict the solution of $x + 9{,}991 > 10{,}000$.

_____

_____

_____

28. **Persevere in Problem Solving** The base of a rectangular prism has a length of 13 inches and a width of $\frac{1}{2}$ inch. The volume of the prism is less than 65 cubic inches. Find all possible heights of the prism. Show your work.

_____

_____

_____

Writing Two-Step Inequalities

? **ESSENTIAL QUESTION**

How do you write a two-step inequality?

EXPLORE ACTIVITY FL Prep for 7.EE.2.4

Modeling Two-Step Inequalities

You can use algebra tiles to model two-step inequalities.

Use algebra tiles to model $2k + 5 \geq -3$.

A Using the line on the mat, draw in the inequality symbol shown in the inequality.

B How can you model the left side of the inequality?

_____
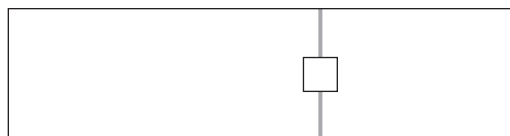
C How can you model the right side of the inequality?

_____

D Use algebra tiles or draw them to model the inequality on the mat.

Reflect

1. **Multiple Representations** How does your model differ from the one you would draw to model the equation $2k + 5 = -3$?

_____

_____

_____

2. Why might you need to change the inequality sign when you solve an inequality using algebra tiles?

_____

_____

_____

Writing Two-Step Inequalities

You can write two-step inequalities to represent real-world problems by translating the words of the problems into numbers, variables, and operations.

EXAMPLE 1 Real World

FL 7.EE.2.4

A mountain climbing team is camped at an altitude of 18,460 feet on Mount Everest. The team wants to reach the 29,029-foot summit within 6 days. Write an inequality to find the average number of feet per day the team must climb to accomplish its objective.

STEP 1 Identify what you are trying to find. This will be the variable in the inequality.

Let d represent the average altitude the team must gain each day.

STEP 2 Identify important information in the problem that you can use to write an inequality.

starting altitude: **18,460 ft**   target altitude: **29,029 ft**
number of days times altitude gained to reach target altitude: $6 \cdot d$

STEP 3 Use words in the problem to tie the information together and write an inequality.

starting altitude	+	number of days	times	altitude gain	is greater than or equal to	target altitude
↓	↓	↓	↓	↓	↓	↓
18,460	+	6	×	d	≥	29,029

$18{,}460 + 6d \geq 29{,}029$

Math Talk
Mathematical Practices

Why is the inequality sign ≥ used, rather than an equal sign?

YOUR TURN

3. The 45 members of the glee club are trying to raise $6,000 so they can compete in the state championship. They already have $1,240. What inequality can you write to find the amount each member must raise, on average, to meet the goal? _____

4. Ella has $40 to spend at the State Fair. Admission is $6 and each ride costs $3. Write an inequality to find the greatest number of rides she can go on.

_____

Writing a Verbal Description of a Two-Step Inequality

You can also write a verbal description to fit a two-step inequality.

EXAMPLE 2   Real World   FL   7.EE.2.4

Write a corresponding real-world problem to represent $2x + 20 \leq 50$.

STEP 1   Analyze what each part of the inequality means mathematically.

x is the solution of the problem, the quantity you are looking for.

2x means that, for a reason given in the problem, the quantity you are looking for is multiplied by 2.

+ 20 means that, for a reason given in the problem, 20 is added to 2*x*.

≤ 50 means that after multiplying the solution *x* by 2 and adding 20 to it, the result can be no greater than 50.

STEP 2   Think of some different situations in which a quantity *x* is multiplied by 2.

You run *x* miles per day for 2 days. So, 2*x* is the total distance run.	You buy 2 items each costing *x* dollars. So, 2*x* is the total cost.

STEP 3   Build on the situation and adjust it to create a verbal description that takes all of the information into account.

- Tomas has run 20 miles so far this week. If he intends to run 50 miles or less, how many miles on average should he run on each of the 2 days remaining in the week?

- Manny buys 2 work shirts that are each the same price. After using a $20 gift card, he can spend no more than $50. What is the maximum amount he can spend on each shirt?

YOUR TURN

Write a real-world problem for each inequality.

5. $3x + 10 > 30$

_____

_____

_____

_____

6. $5x - 50 \leq 100$

_____

_____

_____

_____
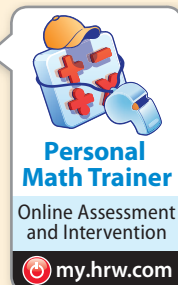
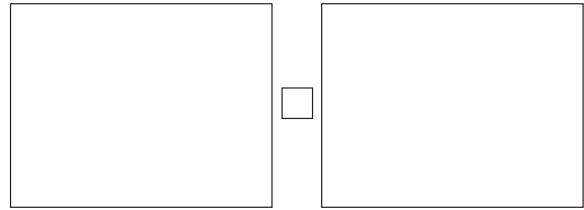**Draw algebra tiles to model each two-step inequality.** (Explore Activity)
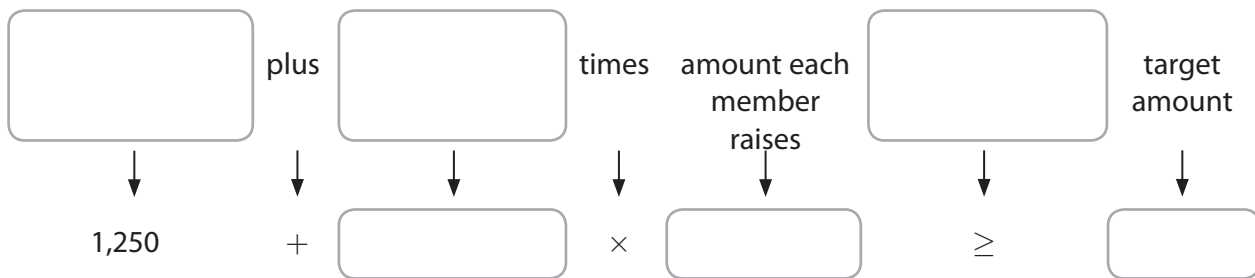
1. $4x - 5 < 7$
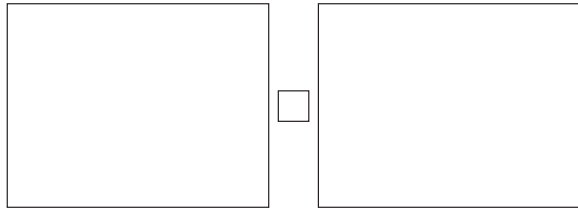
2. $-3x + 6 > 9$

3. The booster club needs to raise at least $7,000 for new football uniforms. So far, they have raised $1,250. Write an inequality to find the average amounts each of the 92 members can raise to meet the club's objective. (Example 1)

Let a represent the amount each member must raise.

amount to be raised: _____

amount already raised: _____

number of members: _____

Use clues in the problem to write an equation.

| | plus | | times | amount each member raises | | target amount |

$1{,}250 \qquad + \qquad \boxed{} \qquad \times \qquad \boxed{} \qquad \geq \qquad \boxed{}$

The inequality that represents the situation is _____.

4. Analyze what each part of $7x - 18 \leq 32$ means mathematically. (Example 2)

x is _____. 		$7x$ is _____.

-18 means that _____.

≤ 32 means that _____

5. Write a real-world problem to represent $7x - 18 \leq 32$.

_____

? ESSENTIAL QUESTION CHECK-IN

6. Describe the steps you would follow to write a two-step inequality you can use to solve a real-world problem.

_____

_____

7.2 Independent Practice

FL 7.EE.2.4

7. Three friends earned more than $200 washing cars. They paid their parents $28 for supplies and divided the rest of money equally. Write an inequality to find possible amounts each friend earned. Identify what your variable represents.

_____

_____

8. Nick has $7.00. Bagels cost $0.75 each, and a small container of cream cheese costs $1.29. Write an inequality to find the numbers of bagels Nick can buy. Identify what your variable represents.

_____

_____

9. Chet needs to buy 4 work shirts, all costing the same amount. After he uses a $25 gift certificate, he can spend no more than $75. Write an inequality to find the possible costs for a shirt. Identify what your variable represents.

_____

_____

10. Due to fire laws, no more than 720 people may attend a performance at Metro Auditorium. The balcony holds 120 people. There are 32 rows on the ground floor, each with the same number of seats. Write an inequality to find the numbers of people that can sit in a ground-floor row if the balcony is full. Identify what your variable represents.

_____

_____

11. Liz earns a salary of $2,100 per month, plus a commission of 5% of her sales. She wants to earn at least $2,400 this month. Write an inequality to find amounts of sales that will meet her goal. Identify what your variable represents.

_____

_____

12. Lincoln Middle School plans to collect more than 2,000 cans of food in a food drive. So far, 668 cans have been collected. Write an inequality to find numbers of cans the school can collect on each of the final 7 days of the drive to meet this goal. Identify what your variable represents.

_____

_____

13. Joanna joins a CD club. She pays $7 per month plus $10 for each CD that she orders. Write an inequality to find how many CDs she can purchase in a month if she spends no more than $100. Identify what your variable represents.

_____

_____

14. Lionel wants to buy a belt that costs $22. He also wants to buy some shirts that are on sale for $17 each. He has $80. What inequality can you write to find the number of shirts he can buy? Identify what your variable represents.

_____

_____

15. Write a situation for $15x - 20 \leq 130$ and solve.

_____

_____

_____

Analyze Relationships **Write $>$, $<$, $\geq$, or $\leq$ in the blank to express the given relationship.**

16. *m* is at least 25   *m* _____ 25

17. *k* is no greater than 9   *k* _____ 9

18. *p* is less than 48   *p* _____ 48

19. *b* is no more than -5 *b* _____ -5

20. *h* is at most 56   *h* _____ 56

21. *w* is no less than 0   *w* _____ 0

22. Critical Thinking Marie scored 95, 86, and 89 on three science tests. She wants her average score for 6 tests to be at least 90. What inequality can you write to find the average scores that she can get on her next three tests to meet this goal? Use *s* to represent the lowest average score.

_____

_____

H.O.T.   **FOCUS ON HIGHER ORDER THINKING**

Work Area

23. Communicate Mathematical Ideas Write an inequality that expresses the reason the lengths 5 feet, 10 feet, and 20 feet could not be used to make a triangle. Explain how the inequality demonstrates that fact.

_____

_____

_____

_____

24. Analyze Relationships The number *m* satisfies the relationship $m < 0$. Write an inequality expressing the relationship between $-m$ and 0. Explain your reasoning.

_____

_____

_____

25. Analyze Relationships The number *n* satisfies the relationship $n > 0$. Write three inequalities to express the relationship between *n* and $\frac{1}{n}$.

_____

LESSON 7.3 Solving Two-Step Inequalities

FL 7.EE.2.4b

Solve...inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set...and interpret it in the context of the problem.

? ESSENTIAL QUESTION

How do you solve a two-step inequality?

Modeling and Solving Two-Step Inequalities

You can solve two-step inequalities using algebra tiles. The method is similar to the one you used to solve two-step equations.

Math On the Spot
my.hrw.com

EXAMPLE 1

FL Prep for 7.EE.2.4b

Use algebra tiles to model and solve $4d - 3 \geq 9$.

STEP 1 Model the inequality. Use a "$\geq$" symbol between the mats.
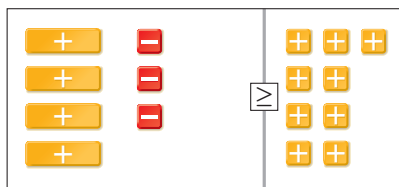
STEP 2 Add three +1 tiles to both sides of the mat.

Math Talk
Mathematical Practices

Why are three +1-tiles added to both sides of the mat in Step 3?
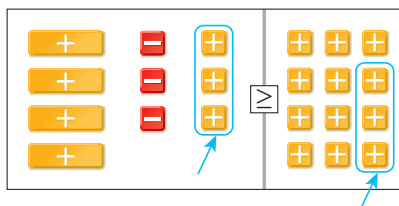
STEP 3 Remove zero pairs from the left side of the mat.
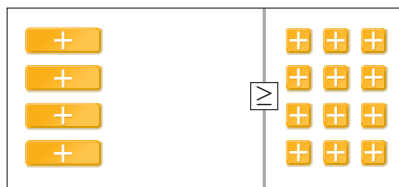
STEP 4 Divide each side into 4 equal groups.

STEP 5 The solution is $d \geq 3$.

Use algebra tiles to model and solve each inequality.

1. $2x + 7 > 11$ _____

2. $5h - 4 \geq 11$ _____

Solving and Interpreting Solutions

You can apply what you know about solving two-step equations and one-step inequalities to solving two-step inequalities.

EXAMPLE 2 Real World   FL   7.EE.2.4b

Serena wants to complete the first 3 miles of a 10-mile run in 45 minutes or less running at a steady pace. The inequality $10 - 0.75p \leq 7$ can be used to find p, the pace, in miles per hour, she can run to reach her goal. Solve the inequality. Then graph and interpret the solution.

STEP 1 Use inverse operations to solve the inequality.

$10 - 0.75p \leq 7$ Subtract 10 from both sides.

$\underline{-10 \qquad\qquad -10}$

$-0.75p \leq -3$

$\dfrac{-0.75p}{-0.75} \geq \dfrac{-3}{-0.75}$ Divide both sides by -0.75.
Reverse the inequality symbol.

$n \geq 4$

STEP 2 Graph the inequality and interpret the circle and the arrow.

Serena can meet her goal by running at a pace of 4 miles per hour.

Serena can run at paces faster than 4 miles per hour and reach her goal.

Serena has to run at a steady pace of at least 4 miles per hour.

3. Joshua wants to complete the first mile of a 5-mile run in 10 minutes or less running at a steady pace. The inequality $5 - \dfrac{p}{6} \leq 4$ can be used to find p, the pace, in miles per hour, he can run to reach his goal. Solve the inequality. Then graph and interpret the solution.
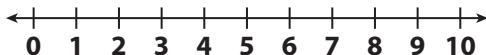
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Determining if a Given Value Makes the Inequality True

You can use substitution to decide whether a given value is the solution of an inequality.

EXAMPLE 3 *Real World* | **FL** 7.EE.2.4b

At Gas 'n' Wash, gasoline sells for $4.00 a gallon and a car wash costs $12. Harika wants to have her car washed and keep her total purchase under $60. The inequality $4g + 12 < 60$ can be used to find g, the number of gallons of gas she can buy. Determine which, if any, of these values is a solution: $g = 10$; $g = 11$; $g = 12$.

STEP 1 Substitute each value for g in the inequality $4g + 12 < 60$.

$g = 10$ $g = 11$ $g = 12$

$4(10) + 12 < 60$ $4(11) + 12 < 60$ $4(12) + 12 < 60$

STEP 2 Evaluate each expression to see if a true inequality results.

$4(10) + 12 \overset{?}{<} 60$ $4(11) + 12 \overset{?}{<} 60$ $4(12) + 12 \overset{?}{<} 60$

$40 + 12 \overset{?}{<} 60$ $44 + 12 \overset{?}{<} 60$ $48 + 12 \overset{?}{<} 60$

$52 \overset{?}{<} 60$ $56 \overset{?}{<} 60$ $60 \overset{?}{<} 60$
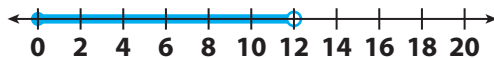
true ✓   true ✓   *not* true ✗

So, Harika can buy 10 or 11 gallons of gas but not 12 gallons.

Check: Solve and graph the inequality.

$4g + 12 < 60$

$4g < 48$

$g < 12$

The closed circle at zero represents the minimum amount she can buy, zero gallons. She cannot buy a negative number of gallons. The open circle at 12 means that she can buy any amount up to but not including 12 gallons.

Math Talk
Mathematical Practices

In Example 2, how will the graph change if Serena's maximum pace is 12 miles per hour?

YOUR TURN

Circle any given values that make the inequality true.

4. $3v - 8 > 22$
$v = 9; v = 10; \; v = 11$
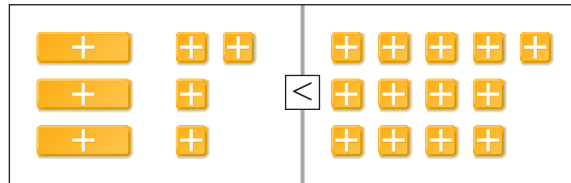
5. $5h + 12 \leq -3$
$h = -3; h = -4; h = -5$

Guided Practice

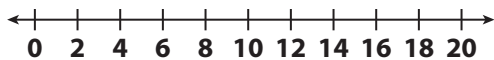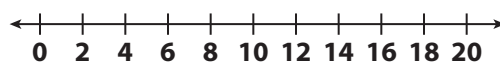**1.** Describe how to solve the inequality $3x + 4 < 13$ using algebra tiles. (Example 1)

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Solve each inequality. Graph and check the solution. (Example 2)

2. $5d - 13 < 32$ _____

0 2 4 6 8 10 12 14 16 18 20

3. $-4b + 9 \leq -7$ _____

0 2 4 6 8 10 12 14 16 18 20

Circle any given values that make the inequality true. (Example 3)

4. $2m + 18 > -4$
$m = -12; m = -11; m = -10$

5. $-6y + 3 \geq 0$
$y = 1; \; y = \frac{1}{2}; y = 0$

0   0.5   1   1.5   2   2.5

6. Lizzy has 6.5 hours to tutor 4 students and spend 1.5 hours in a lab. She plans to tutor each student the same amount of time. The inequality $6.5 - 4t \geq 1.5$ can be used to find t, the amount of time in hours Lizzy could spend with each student. Solve the inequality. Graph and interpret the solution. Can Lizzy tutor each student for 1.5 hours? Explain. (Examples 2 and 3)

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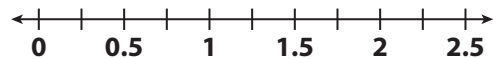
? ESSENTIAL QUESTION CHECK-IN

7. How do you solve a two-step inequality?

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7.3 Independent Practice

FL 7.EE.2.4b

Solve each inequality. Graph and check the solution.

8. $2s + 5 \geq 49$ _____
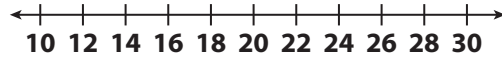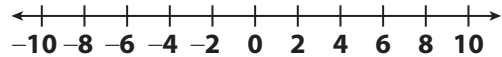
10 12 14 16 18 20 22 24 26 28 30

9. $-3t + 9 \geq -21$ _____

−10 −8 −6 −4 −2 0 2 4 6 8 10

10. $55 > -7v + 6$ _____

−10 −9 −8 −7 −6 −5 −4 −3 −2 −1 0

11. $41 > 6m - 7$ _____

0 1 2 3 4 5 6 7 8 9 10

12. $\dfrac{a}{-8} + 15 > 23$ _____

−70 −69 −68 −67 −66 −65 −64 −63 −62 −61 −60

13. $\dfrac{f}{2} - 22 < 48$ _____

100 105 110 115 120 125 130 135 140 145 150

14. $-25 + \dfrac{t}{2} \geq 50$ _____

130 135 140 145 150 155 160 165 170 175 180

15. $10 + \dfrac{g}{-9} > 12$ _____

−20 −19 −18 −17 −16 −15 −14 −13 −12 −11 −10

16. $25.2 \leq -1.5y + 1.2$ _____

−20 −19 −18 −17 −16 −15 −14 −13 −12 −11 −10

17. $-3.6 \geq -0.3a + 1.2$ _____

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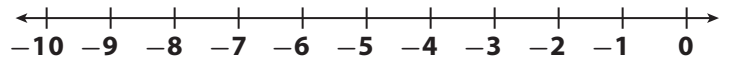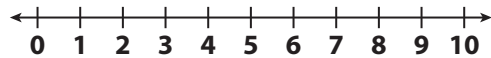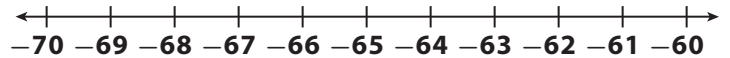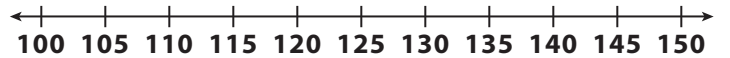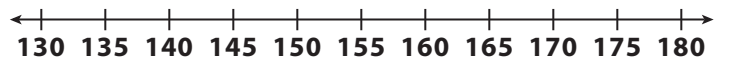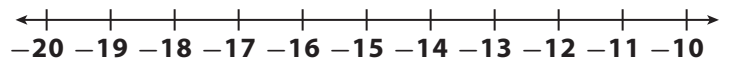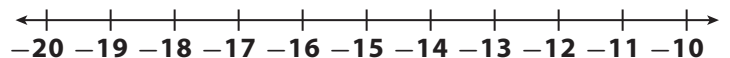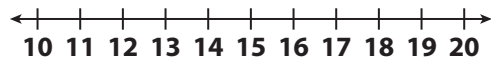
18. What If? The perimeter of a rectangle is at most 80 inches. The length of the rectangle is 25 inches. The inequality $80 - 2w \geq 50$ can be used to find w, the width of the rectangle in inches. Solve the inequality and interpret the solution. How will the solution change if the width must be at least 10 inches and a whole number?

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19. Interpret the Answer Grace earns $7 for each car she washes. She always saves $25 of her weekly earnings. This week, she wants to have at least $65 in spending money. How many cars must she wash? Write and solve an inequality to represent this situation. Interpret the solution in context.

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Work Area

20. Critical Thinking Is there any value of x with the property that $x < x - 1$? Explain your reasoning.
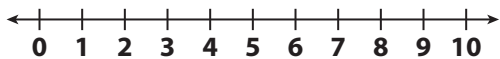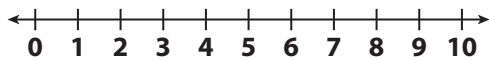
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21. Analyze Relationships A *compound inequality* consists of two simple equalities joined by the word "*and*" or "*or*." Graph the solution sets of each of these compound inequalities.

a. $x > 2$ and $x < 7$

b. $x < 2$ or $x > 7$

c. Describe the solution set of the compound inequality $x < 2$ and $x > 7$.

_____

d. Describe the solution set of the compound inequality $x > 2$ or $x < 7$.

_____

22. Communicate Mathematical Ideas Joseph used the problem-solving strategy Work Backward to solve the inequality $2n + 5 < 13$. Shawnee solved the inequality using the algebraic method you used in this lesson. Compare the two methods.

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Ready to Go On?

7.1 Writing and Solving One-Step Inequalities

Solve each inequality.

1. $n + 7 < -3$ _____

2. $5p \geq -30$ _____

3. $14 < k + 11$ _____

4. $\frac{d}{-3} \leq -6$ _____

5. $c - 2.5 \leq 2.5$ _____

6. $12 \geq -3b$ _____

7. Jose has scored 562 points on his math tests so far this semester. To get an A for the semester, he must score at least 650 points. Write and solve an inequality to find the minimum number of points he must score on the remaining tests in order to get an A.

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7.2 Writing Two-Step Inequalities

8. During a scuba dive, Lainey descended to a point 20 feet below the ocean surface. She continued her descent at a rate of 20 feet per minute. Write an inequality you could solve to find the number of minutes she can continue to descend if she does not want to reach a point more than 100 feet below the ocean surface.

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7.3 Solving Two-Step Inequalities

Solve.

9. $2s + 3 > 15$ _____

10. $-\frac{d}{12} - 6 < 1$ _____

11. $-6w - 18 \geq 36$ _____

12. $\frac{z}{4} + 22 \leq 38$ _____

13. $\frac{b}{9} - 34 < -36$ _____

14. $-2p + 12 > 8$ _____

? ESSENTIAL QUESTION

15. How can you recognize whether a real-world situation should be represented by an equation or an inequality?
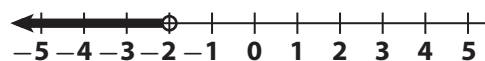
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Assessment Readiness

Selected Response

1. Which graph models the solution of the inequality $-6 \leq -3x$?

Ⓐ $-5\ -4\ -3\ -2\ -1\quad 0\quad 1\quad 2\quad 3\quad 4\quad 5$

Ⓑ $-5\ -4\ -3\ -2\ -1\quad 0\quad 1\quad 2\quad 3\quad 4\quad 5$

Ⓒ $-5\ -4\ -3\ -2\ -1\quad 0\quad 1\quad 2\quad 3\quad 4\quad 5$

Ⓓ $-5\ -4\ -3\ -2\ -1\quad 0\quad 1\quad 2\quad 3\quad 4\quad 5$

2. A taxi cab costs $1.75 for the first mile and $0.75 for each additional mile. You have $20 to spend on your ride. Which inequality could be solved to find how many miles you can travel, if n is the number of additional miles?

Ⓐ $1.75n + 0.75 \geq 20$

Ⓑ $1.75n + 0.75 \leq 20$

Ⓒ $0.75n + 1.75 \geq 20$

Ⓓ $0.75n + 1.75 \leq 20$

3. The inequality $\frac{9}{5}C + 32 < -40$ can be used to find Celsius temperatures that are less than $-40°$ Fahrenheit. What is the solution of the inequality?

Ⓐ $C < 40$ Ⓒ $C < -40$

Ⓑ $C < -\frac{40}{9}$ Ⓓ $C < -\frac{72}{5}$

4. The 30 members of a choir are trying to raise at least $1,500 to cover travel costs to a singing camp. They have already raised $600. Which inequality could you solve to find the average amounts each member can raise that will at least meet the goal?

Ⓐ $30x + 600 > 1,500$

Ⓑ $30x + 600 \geq 1,500$

Ⓒ $30x + 600 < 1,500$

Ⓓ $30x + 600 \leq 1,500$

5. Which represents the solution for the inequality $3x - 7 > 5$?

Ⓐ $x < 4$ Ⓒ $x > 4$

Ⓑ $x \leq 4$ Ⓓ $x \geq 4$

6. Which inequality has the following graphed solution?

$-5\ -4\ -3\ -2\ -1\quad 0\quad 1\quad 2\quad 3\quad 4\quad 5$

Ⓐ $3x + 8 \leq 2$ Ⓒ $2x + 5 \leq 1$

Ⓑ $4x + 12 < 4$ Ⓓ $3x + 6 < 3$

7. Divide: $-36 \div 6$.

Ⓐ 30 Ⓒ -6

Ⓑ 6 Ⓓ -30

8. Eleni bought 2 pounds of grapes at a cost of $3.49 per pound. She paid with a $10 bill. How much change did she get back?

Ⓐ $3.02   Ⓒ $6.51

Ⓑ $4.51   Ⓓ $6.98

Mini-Task

9. In golf, the lower your score, the better. Negative scores are best of all. Teri scored $+1$ on each of the first three holes at a nine-hole miniature golf course. Her goal is a total score of -9 or better after she has completed the final six holes.

a. Let h represent the score Teri must average on each of the last six holes in order to meet her goal. Write a two-step inequality you can solve to find h.

_____

b. Solve the inequality.

_____

Study Guide Review

MODULE 6 Expressions and Equations

Key Vocabulary
algebraic expression *(expresión algebraica)*
equation *(ecuación)*

? ESSENTIAL QUESTION

How can you use equations to solve real-world problems?

EXAMPLE 1

Huang and Belita both repair computers. Huang makes $50 a day plus $25 per repair. Belita makes $20 a day plus $35 per repair. Write an expression for Huang and Belita's total daily earnings if they make the same number of repairs r.

Huang: $50 + 25r$

Belita: $20 + 35r$

Together: $(50 + 25r) + (20 + 35r) = 50 + 20 + 25r + 35r$

$$= 70 + 60r$$

Huang and Belita earn $70 + 60r$ together.

EXAMPLE 2

A skydiver's parachute opens at a height of 2,790 feet. He then falls at a rate of $-15\frac{1}{2}$ feet per second. How long will it take the skydiver to reach the ground?

Let x represent the number of seconds it takes to reach the ground.

$$-15\frac{1}{2}x = -2{,}790$$

$$-\frac{31}{2}x = -2{,}790 \qquad \text{Write as a fraction.}$$

$$\left(-\frac{2}{31}\right)\left(-\frac{31}{2}x\right) = \left(-\frac{2}{31}\right)(-2{,}790) \qquad \text{Multiply both sides by the reciprocal.}$$

$$x = 180$$

It takes 180 seconds for the skydiver to reach the ground.

EXAMPLE 3

A clothing store sells clothing for 2 times the wholesale cost plus $10. The store sells a pair of pants for $48. How much did the store pay for the pants? Represent the solution on a number line.

Let w represent the wholesale cost of the pants, or the price paid by the store.

$2w + 10 = 48$

$\qquad 2w = 38$ *Subtract 10 from both sides.*

$\qquad \ \ w = 19$ *Divide both sides by 2.*

10 11 12 13 14 15 16 17 18 19 20
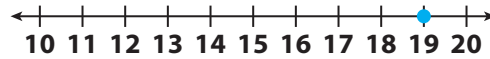
The store paid $19 for the pants.

EXERCISES

Simplify each expression. (Lesson 6.1)

1. $(2x + 3\frac{2}{5}) + (5x - \frac{4}{5})$ _____

2. $(-0.5x - 4) - (1.5x + 2.3)$ _____

3. $9(3t + 4b)$ _____

4. $0.7(5a - 13p)$ _____

Factor each expression. (Lesson 6.1)

5. $8x + 56$ _____

6. $3x + 57$ _____

Use inverse operations to solve each equation. (Lesson 6.2)

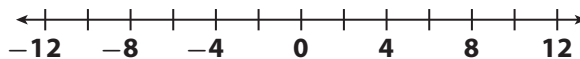**7.** $1.6 + y = -7.3$ _____

8. $-\frac{2}{3}n = 12$ _____

9. The cost of a ticket to an amusement park is $42 per person. For groups of up to 8 people, the cost per ticket decreases by $3 for each person in the group. Marcos's ticket cost $30. Write and solve an equation to find the number of people in Marcos's group. (Lesson 6.3, 6.4)

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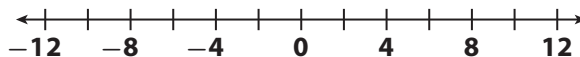**Solve each equation. Graph the solution on a number line.** (Lesson 6.4)

10. $8x - 28 = 44$

−12 −8 −4 0 4 8 12

_____

11. $-5z + 4 = 34$

−12 −8 −4 0 4 8 12

_____

? ESSENTIAL QUESTION

How can you use inequalities to solve real-world problems?

EXAMPLE 1

Amy is having her birthday party at a roller skating rink. The rink charges a fee of $50 plus $8 per person. If Amy wants to spend at most $170 for the party at the rink, how many people can she invite to her party?

Let p represent the number of people skating at the party.

$50 + 8p \leq 170$

$\qquad 8p \leq 120$ *Subtract 50 from both sides.*

$\qquad \dfrac{8p}{8} \leq \dfrac{120}{8}$ *Divide both sides by 8.*

$\qquad p \leq 15$

Up to 15 people can skate, so Amy can invite up to 14 people to her party.

EXAMPLE 2

Determine which, if any, of these values makes the inequality $-7x + 42 \leq 28$ true: $x = -1, x = 2, x = 5$.

$-7(-1) + 42 \leq 28 \qquad -7(2) + 42 \leq 28 \qquad -7(5) + 42 \leq 28$

$x = 2$ and $x = 5$

Substitute each value for x in the inequality and evaluate the expression to see if a true inequality results.

EXERCISES

1. Prudie needs $90 or more to be able to take her family out to dinner. She has already saved $30 and wants to take her family out to eat in 4 days. **(Lesson 7.2)**

   a. Suppose that Prudie earns the same each day. Write an inequality to find how much she needs to earn each day.

   _____
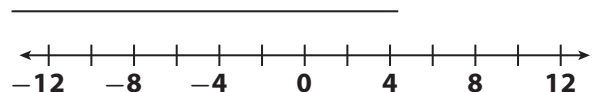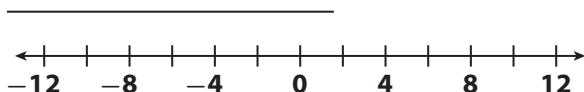
   b. Suppose that Prudie earns $18 each day. Will she have enough money to take her family to dinner in 4 days? Explain.

   _____

**Solve each inequality. Graph and check the solution. (Lesson 7.3)**

2. $11 - 5y < -19$

   _____

3. $7x - 2 \leq 61$

   _____

1. **CAREERS IN MATH** **Mechanical Engineer** A mechanical engineer is testing the amount of force needed to make a spring stretch by a given amount. The force y is measured in units called *Newtons*, abbreviated N. The stretch x is measured in centimeters. Her results are shown in the graph.

Spring Stretch
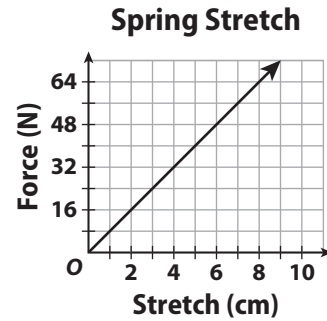
a. Write an equation for the line. Explain, using the graph and then using the equation, why the relationship is proportional.

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_____

b. Identify the rate of change and the constant of proportionality.

_____

c. What is the meaning of the constant of proportionality in the context of the problem?

_____

d. The engineer applies a force of 41.6 Newtons to the spring. Write and solve an equation to find the corresponding stretch in the spring.

_____

2. A math tutor charges $30 for a consultation, and then $25 per hour. An online tutoring service charges $30 per hour.

a. Does either service represent a proportional relationship? Explain.
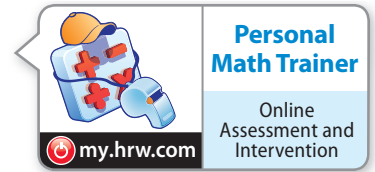
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b. Write an equation for the cost c of h hours of tutoring for either service. Which service charges less for 4 hours of tutoring? Show your work.

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Assessment Readiness

Selected Response

1. Which expression is equivalent to $(9x - 3\frac{1}{8}) - (7x + 1\frac{3}{8})$?

Ⓐ $2x - 4\frac{1}{2}$ Ⓒ $2x - 1\frac{3}{4}$

Ⓑ $16x - 4\frac{1}{2}$ Ⓓ $16x - 1\frac{3}{4}$

2. Timothy began the week with $35. He bought lunch at school, paying $2.25 for each meal. Let x be the number of meals he bought at school and y be the amount of money he had left at the end of the week. Which equation represents the relationship in the situation?

Ⓐ $y = 2.25x + 35$

Ⓑ $y = 35 - 2.25x$

Ⓒ $x = 35 - 2.25y$

Ⓓ $y = 2.25x - 35$

3. Which expression factors to $8(x + 2)$?

Ⓐ $8x + 2$ Ⓒ $16x$

Ⓑ $8x + 10$ Ⓓ $8x + 16$

4. Ramón's toll pass account has a value of $32. Each time he uses the toll road, $1.25 is deducted from the account. When the value drops below $10, he must add value to the toll pass. Which inequality represents how many times Ramón can use the toll road without having to add value to the toll pass?

Ⓐ $10 - 1.25t \geq 0$

Ⓑ $-1.25t + 32 < 10$

Ⓒ $32 - 1.25t \geq 10$

Ⓓ $32 - 10t \geq 1.25$

5. A taxi costs $1.65 for the first mile and $0.85 for each additional mile. Which equation could be solved to find the number x of additional miles traveled in a taxi given that the total cost of the trip is $20?

Ⓐ $1.65x + 0.85 = 20$

Ⓑ $0.85x + 1.65 = 20$

Ⓒ $1.65x - 0.85 = 20$

Ⓓ $0.85x - 1.65 = 20$

6. A sales tax of 6% is added to the price of an item. If Marisa buys an item, which expression indicates how much she will pay in all?

Ⓐ $n + 0.06$ Ⓒ $n + 0.06n$

Ⓑ $0.06n$ Ⓓ $0.06 + 0.06n$

7. Which equation has the solution $x = 12$?

Ⓐ $4x + 3 = 45$

Ⓑ $3x + 6 = 42$

Ⓒ $2x - 5 = 29$

Ⓓ $5x - 8 = 68$

8. The 23 members of the school jazz band are trying to raise at least $1,800 to cover the cost of traveling to a competition. The members have already raised $750. Which inequality could you solve to find the amount that each member should raise to meet the goal?

Ⓐ $23x + 750 > 1,800$

Ⓑ $23x + 750 \geq 1,800$

Ⓒ $23x + 750 < 1,800$

Ⓓ $23x + 750 \leq 1,800$

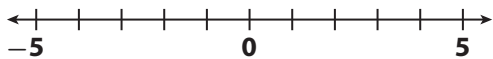**9.** What is the solution of the inequality $2x - 9 < 7$?

   (A) $x < 8$

   (B) $x \leq 8$

   (C) $x > 8$

   (D) $x \geq 8$

10. Which inequality has the solution $n < 5$?

   (A) $4n + 11 > -9$

   (B) $4n + 11 < -9$

   (C) $-4n + 11 < -9$

   (D) $-4n + 11 > -9$

11. Which inequality has the solution shown?

   (A) $3x + 5 < 2$

   (B) $4x + 12 < 4$

   (C) $2x + 5 \leq 1$

   (D) $3x + 6 \leq 3$

12. On a $4\frac{1}{2}$ hour trip, Leslie drove $\frac{2}{3}$ of the time. For how many hours did Leslie drive?

   (A) 3 hours

   (B) $3\frac{1}{2}$ hours

   (C) $3\frac{2}{3}$ hours

   (D) $3\frac{5}{6}$ hours

13. During a sale, the price of a sweater was changed from $20 to $16. What was the percent of decrease in the price of the sweater?

   (A) 4%

   (B) 20%

   (C) 25%

   (D) 40%

Mini-Task

14. Max wants to buy some shorts that are priced at $8 each. He decided to buy a pair of sneakers for $39, but the total cost of the shorts and the sneakers must be less than $75.

  a. Write an inequality to find out how many pairs of shorts Max can buy.

  b. Suppose that Max wants to buy 6 pairs of shorts. Will he have enough money? Explain.

  c. Solve the inequality to find the greatest number of pairs of shorts that Max can buy. Show your work.